# Charged tracer particle in a diffusive environment

A mock-up multiphysics model

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Ingredients:

1. Two dimensional diffusion equation for density field $\rho(\mathbf{r}, t)$ governed by PDE

$$\frac{\partial \rho}{\partial t} = D\left(\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2}\right). \tag{1}$$

2. Tracer particle satisfies ODE

$$m\ddot{\mathbf{R}} = \mathbf{F} - \alpha(\rho)\dot{\mathbf{R}}, \tag{2a}$$

where $m$ is the mass, $\mathbf{F}$ is a force acting on the particle and the friction constant $\alpha$ is proportional to the viscosity, and is density dependent.

   - Ad hoc form for density dependent friction constant $\alpha$:

$$\alpha(\rho) = \alpha_0(1 + a\rho). \tag{2b}$$

   - Ad hoc form for force, like a constant electric field:

$$\mathbf{F} = qE\hat{\mathbf{x}}. \tag{2c}$$

3. And let's say we have periodic boundary conditions in all directions, i.e.,

$$\mathbf{r} \sim \mathbf{r} + L(k\hat{\mathbf{x}} + l\hat{\mathbf{y}}). \tag{3}$$

where $L$ is the length of the side of the square periodic box, and $k$ and $l$ are integers. Positional coordinates can thus be restricted to lie between $-L/2$ and $L/2$.

4. Initial conditions:

$$\rho(\mathbf{r}) = \exp\left[-\frac{\|\mathbf{r}\|^2}{2\sigma_0^2}\right], \tag{4a}$$

$$\mathbf{R}(0) = \mathbf{R}_0, \tag{4b}$$

$$\dot{\mathbf{R}}(0) = \mathbf{V}_0. \tag{4c}$$

5. Values for the parameters (arbitrary) are

$$D = 1;\ m = 1;\ \alpha_0 = 1; a = 15;\ qE = 1;\ L = 10;\ \sigma_0 = 1;\ \mathbf{R}_0 = \mathbf{0};\ \mathbf{V}_0 = 10\hat{\mathbf{y}} \tag{5}$$