A basic introduction to Complex Networks, using python

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May 14th, 2015

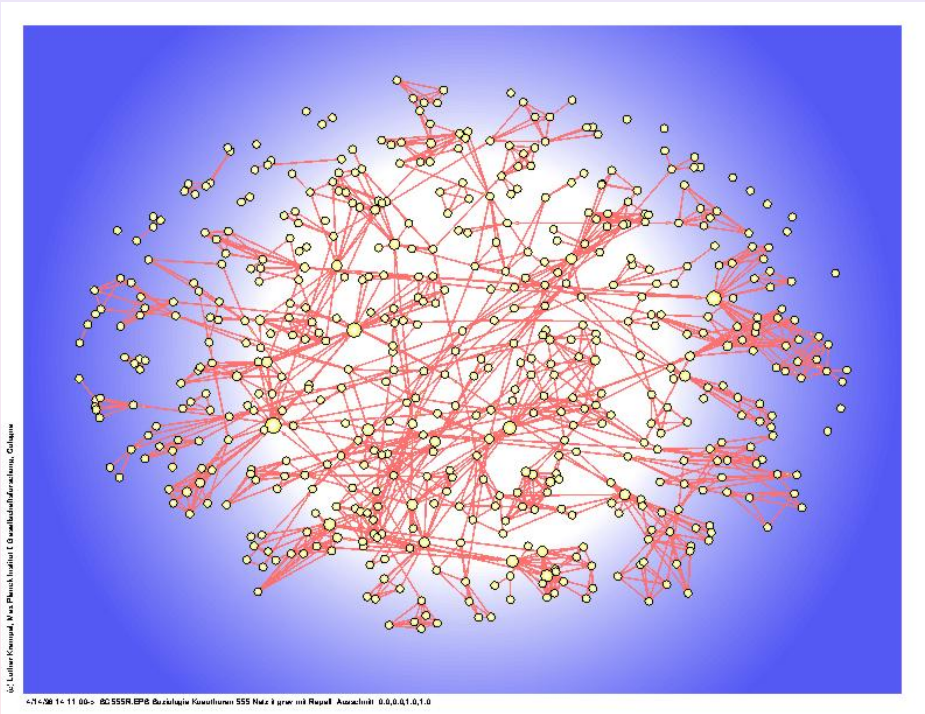
Outline

Examples of Complex Networks

A system composed of many nonlinear interacting units often forms a complex system with new emergent properties that are not held by the individual units.

Examples:

- circadian rhythms,
- electrochemical reactions,
- laser arrays,
- neuron networks,
- populations of fireflies,
- Josephson junction arrays..., etc.



▲ Co-Authorship Network By Lothar Krempel

http://www.mpi-fg-koeln.mpg.de/~lk/netvis/Huge.html

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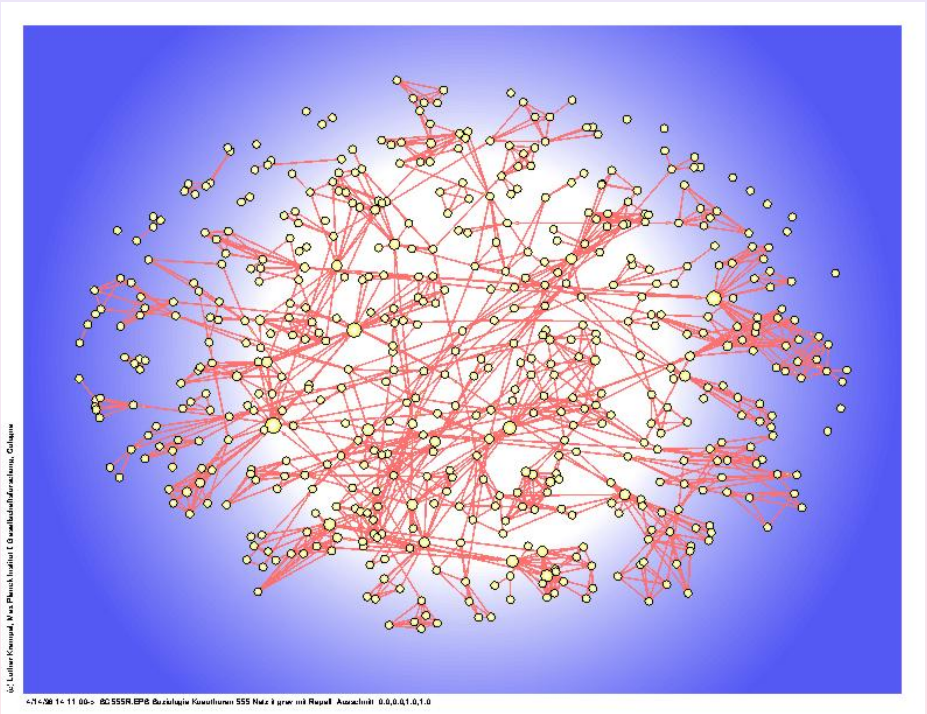

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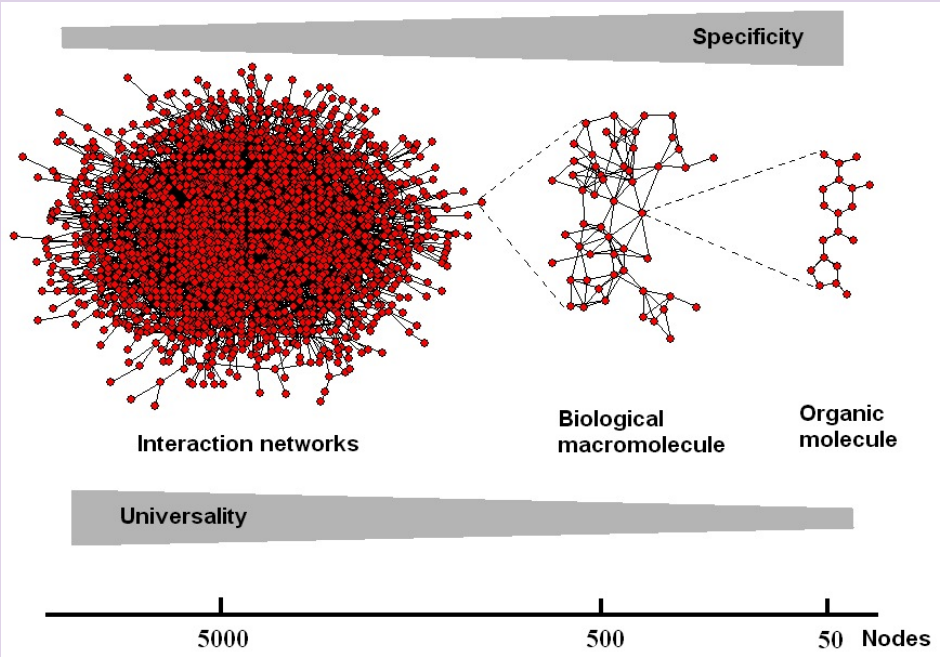
Examples of Complex Networks

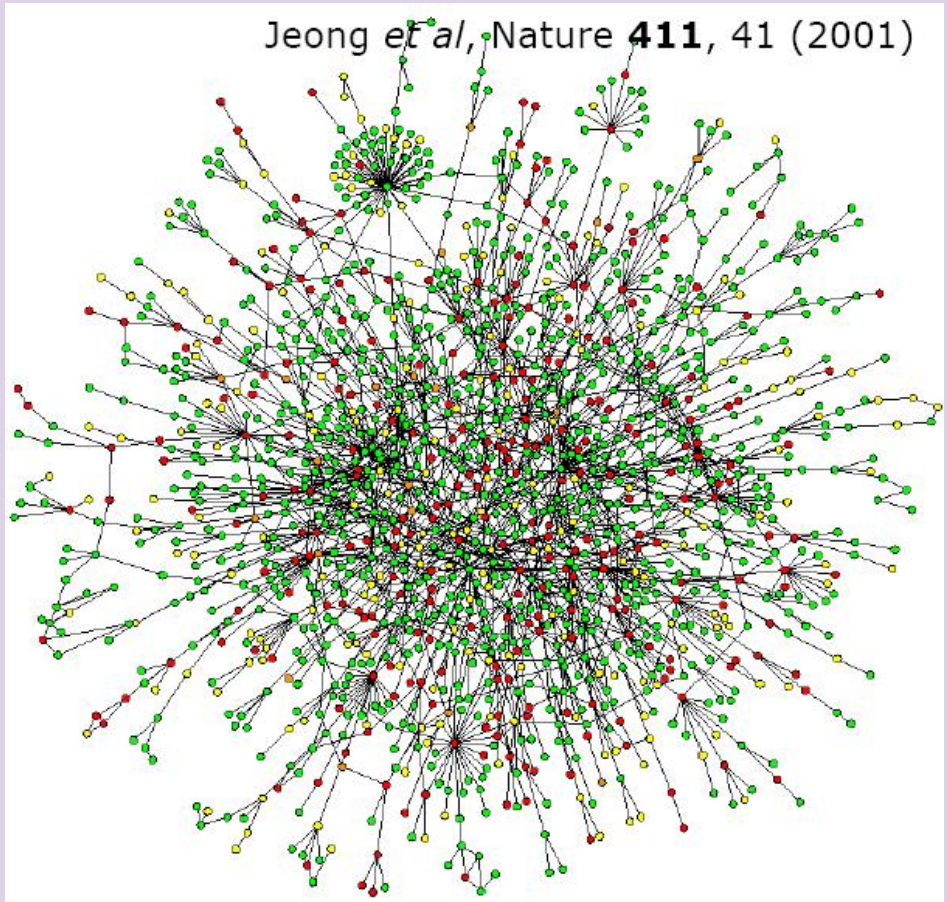
Biological networks...

- Proteins, molecules, chemical reactions...



Map of protein-protein interactions. The colour of a node signifies the phenotypic effect of removing the corresponding protein (lethal; non-lethal; slow growth; unknown). ▸

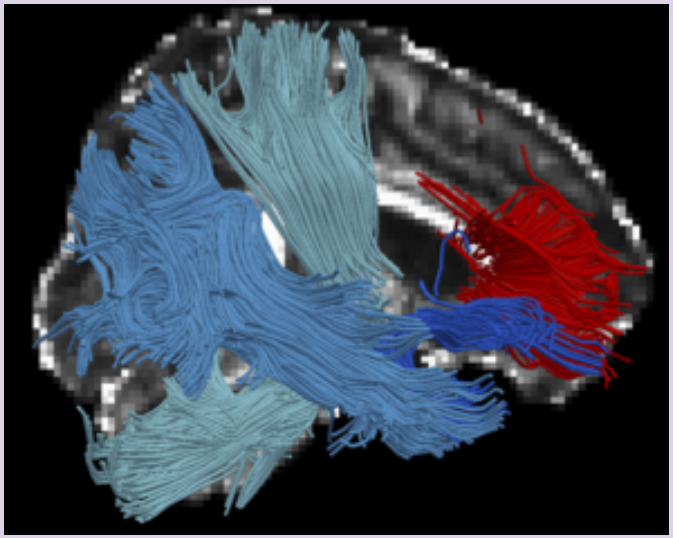
- Genes, proteins, ...



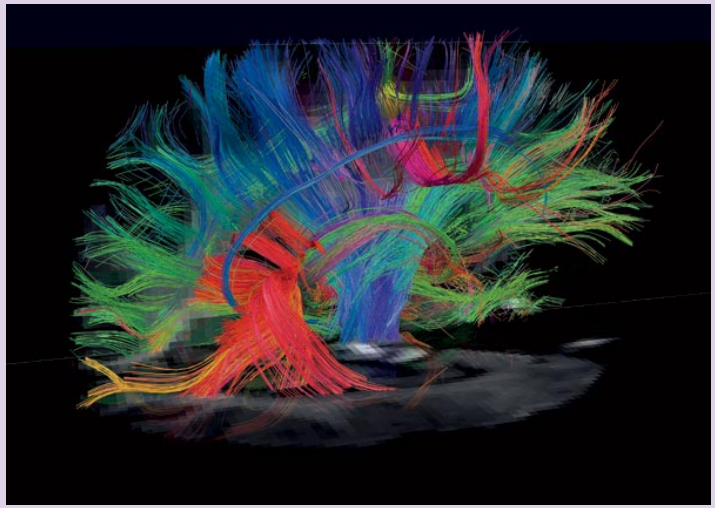
Examples of Complex Networks

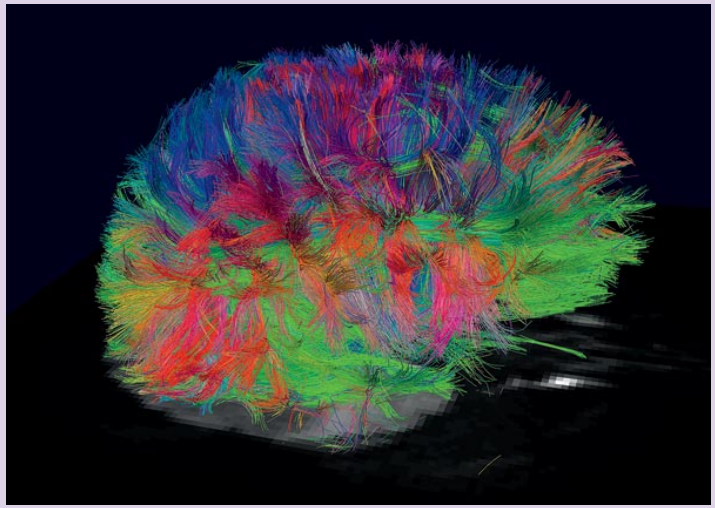
Brain networks...

Physiological networks



Neurological pathways

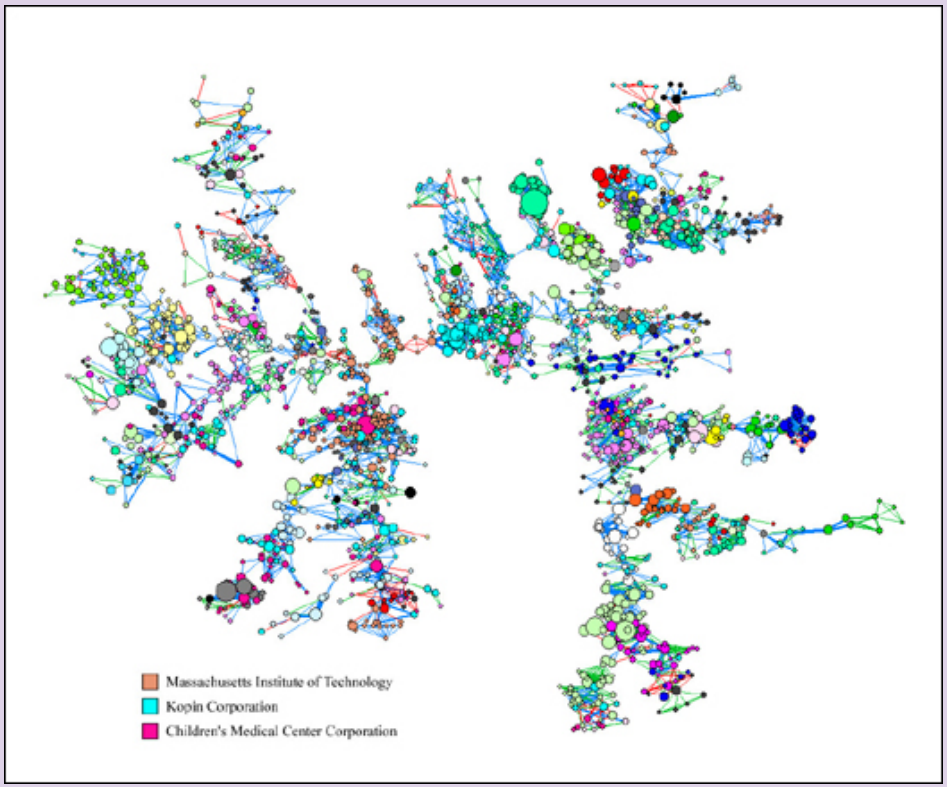




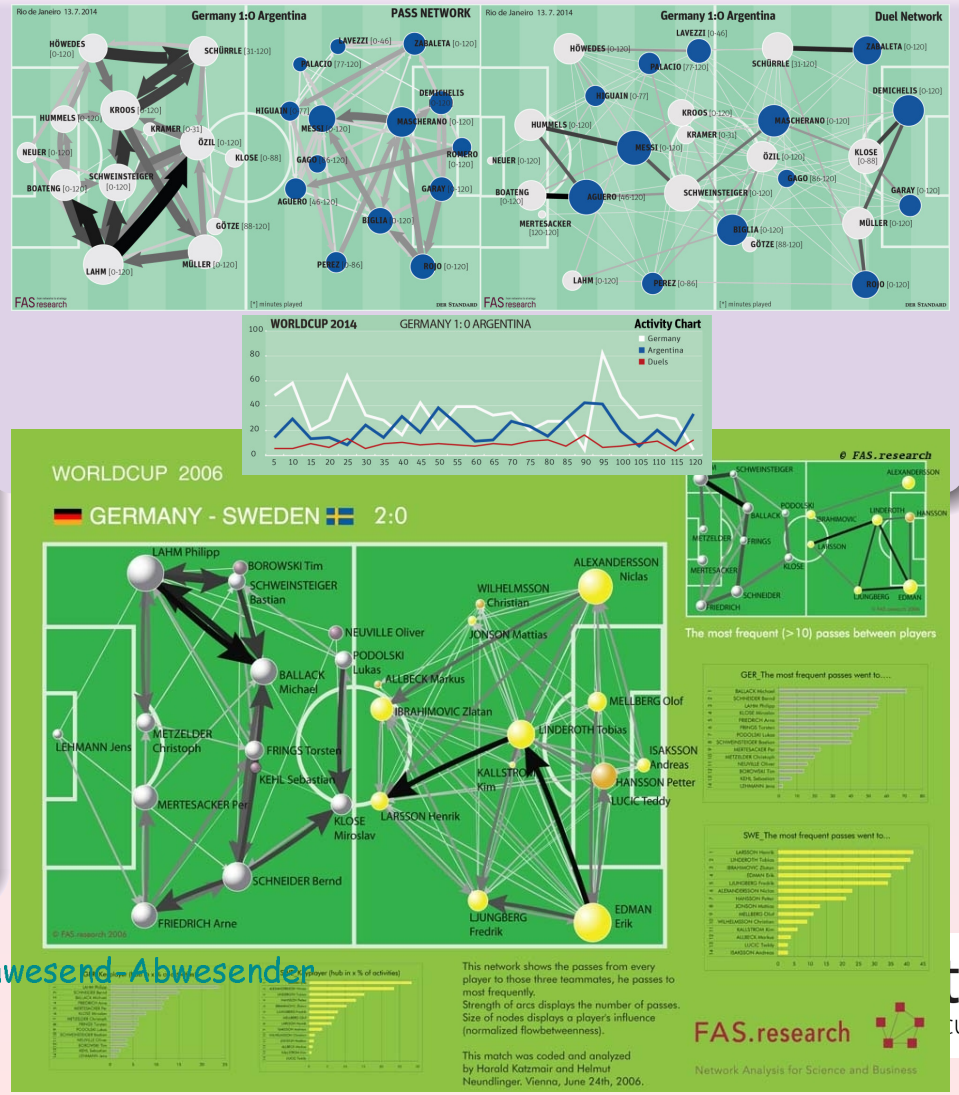
Examples of Complex Networks

Social networks...

- Boston Inventor Networks mid-1990s



- Sports: Futbol - world cup...

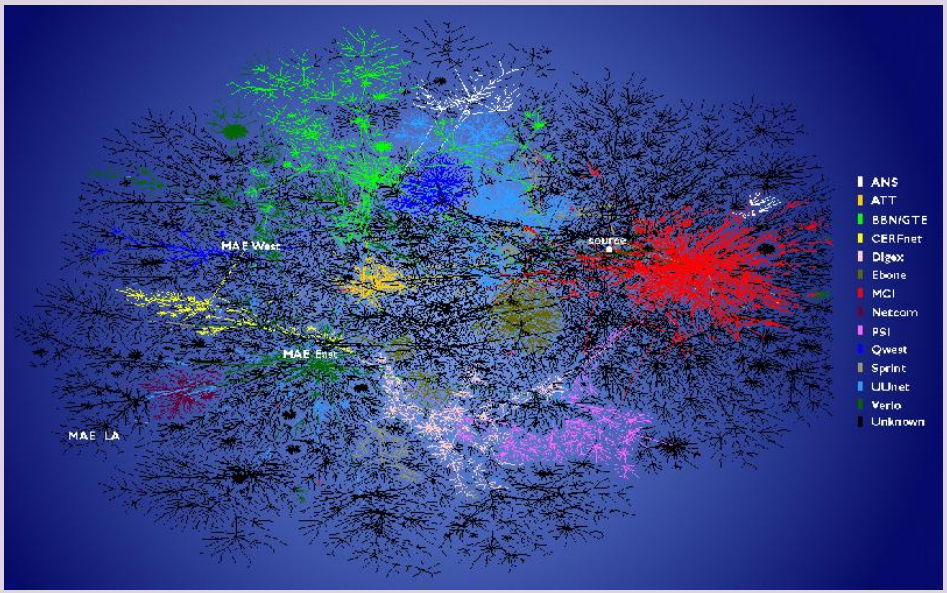


http://derstandard.at/2000002589338/Messi-glaenzt-als-anwesend-Abwesender

Examples of Complex Networks

Internet

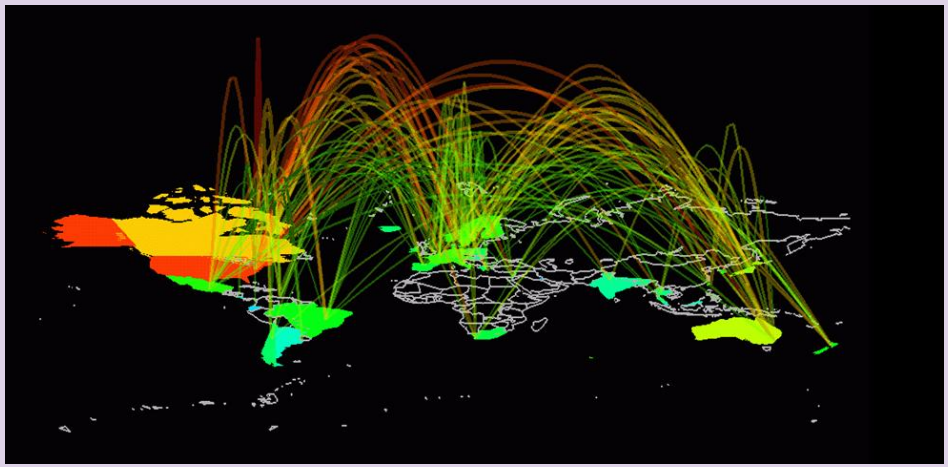
- Internet connectivity,



with selected backbone ISPs (Internet Service Provider)

colored separately

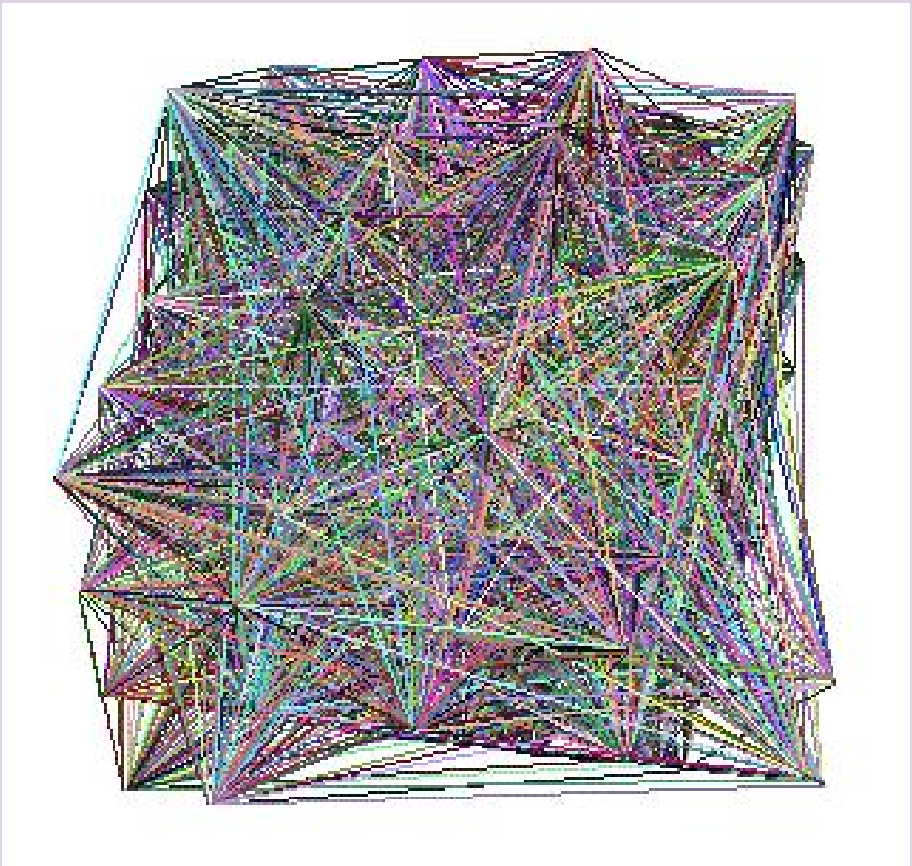
- Arc map showing the world-wide internet traffic



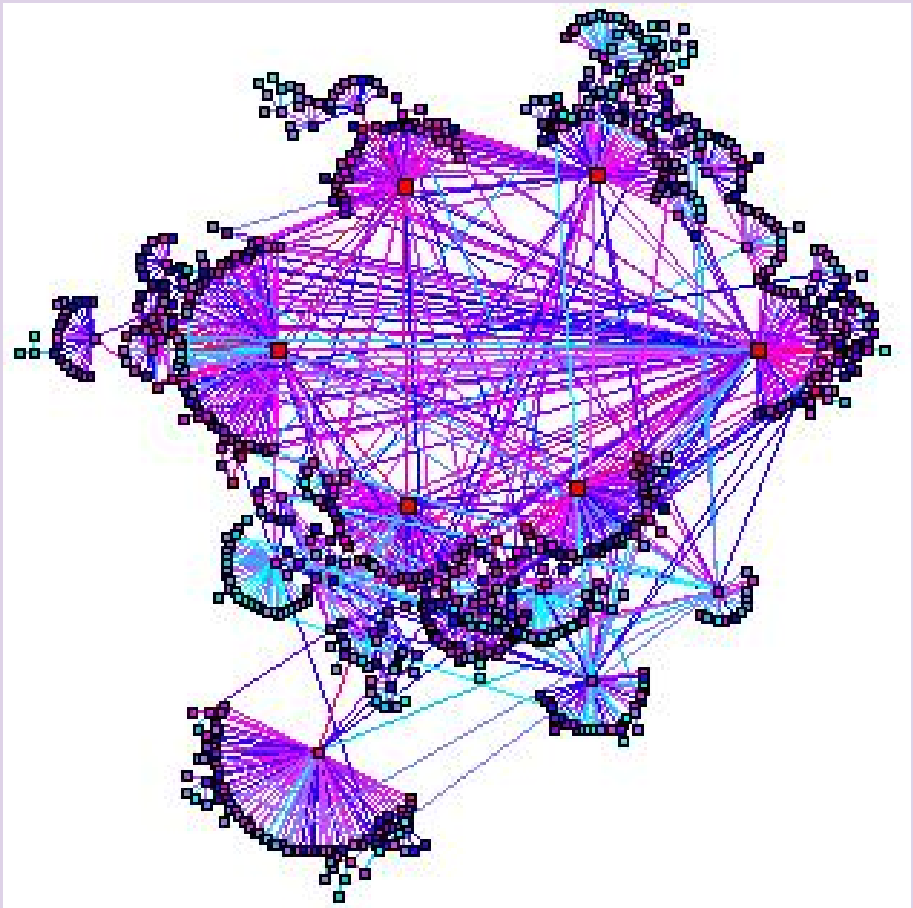
Examples of Complex Networks

Internet

- Mapping the internet: tracing domain routes

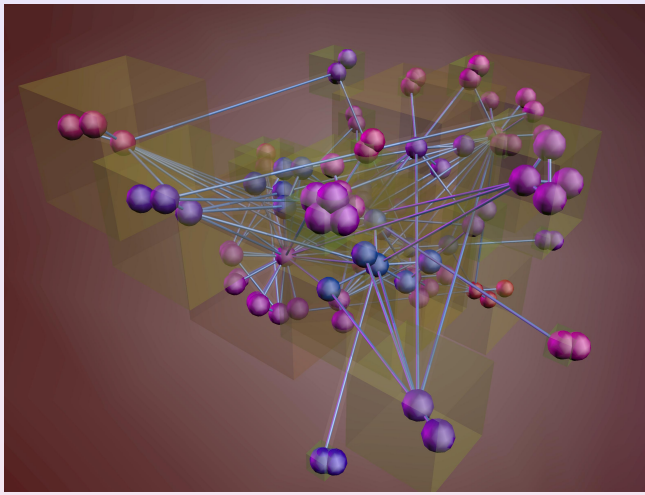


- Hierarchical topology of the international web cache



nodes, network, maps...

A network is a set of nodes interconnected via links.



1. [nodes]: ensemble of N oscillators
2. [dynamics+network]: each oscillator evolves according to its own intrinsic dynamics [map (fn)] + a coupling term [network].
3. The coupling term is defined by:
   - coupling strength
   - coupling topology (all-to-all, n-n, etc)

Nodes and Links can be anything depending on the **context**

from the previous examples:

- Internet: Nodes $\rightsquigarrow$ routers & Links $\rightsquigarrow$ optical fibers.
- WWW: Nodes $\rightsquigarrow$ document files & Links $\rightsquigarrow$ hyperlinks.
- The Scientific Citation Network: Nodes $\rightsquigarrow$ papers & Links $\rightsquigarrow$ citations.
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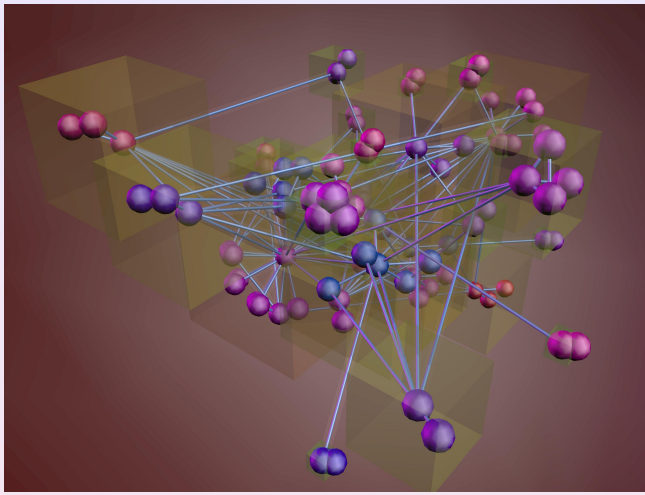

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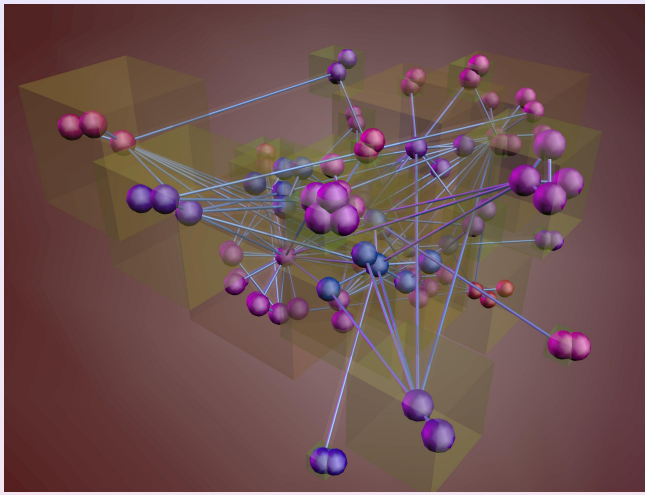

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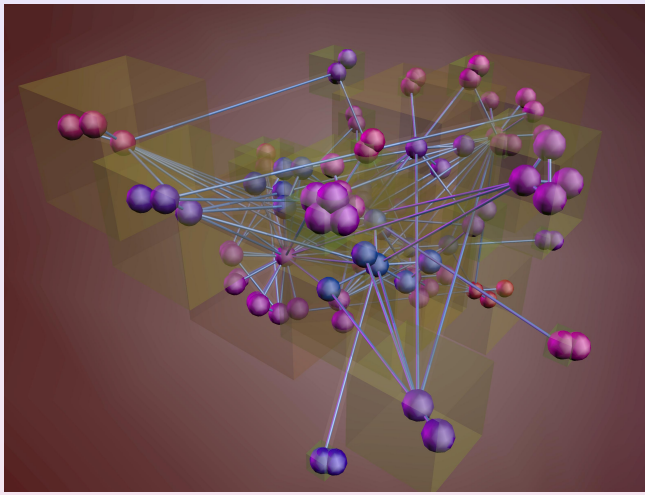

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- **networkX**: http://networkx.lanl.gov/
- **PyCX** Project: http://pycx.sf.net/

- ComplexNetworkSim
- SimPy
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Defining a graph

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g.add_edge('b','c',weight=1.5)
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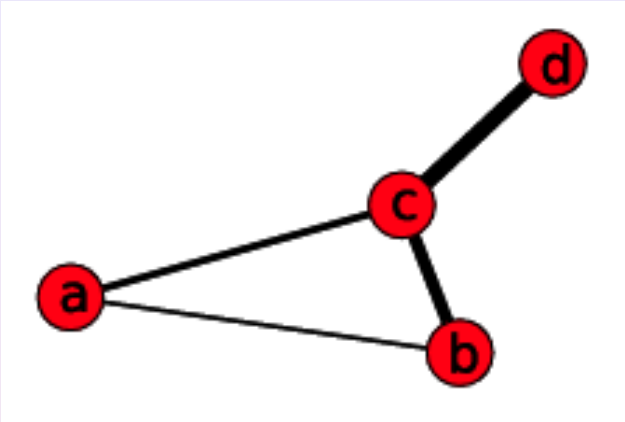

Some basic props. of the graph

```
g.number_of_nodes()  # also g.order()
4
g.order()  # also g.size()
4
```

```
g.nodes()
['a', 'c', 'b', 'd']
g.edges()
[('a', 'c'), ('a', 'b'), ('c', 'b'), ('c', 'd')]

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```

```
g.degree('a')
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g.neighbors('c')
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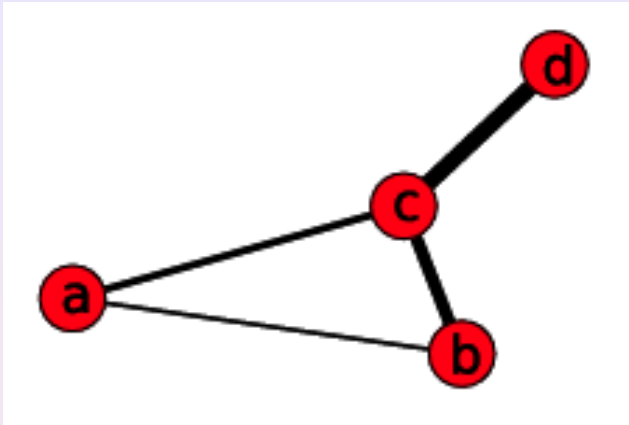

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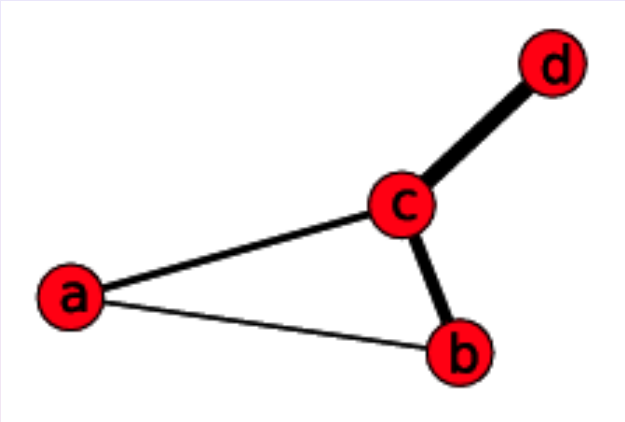

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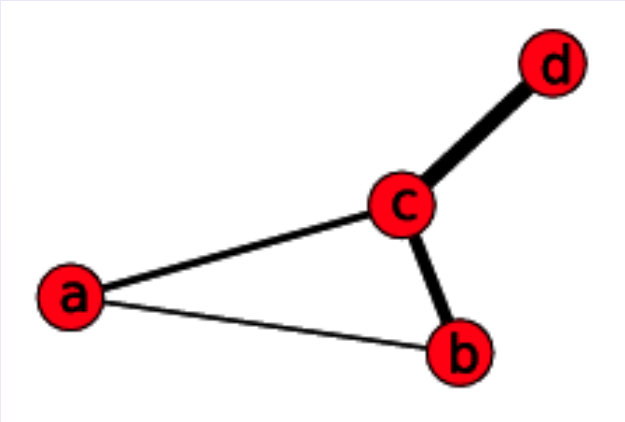

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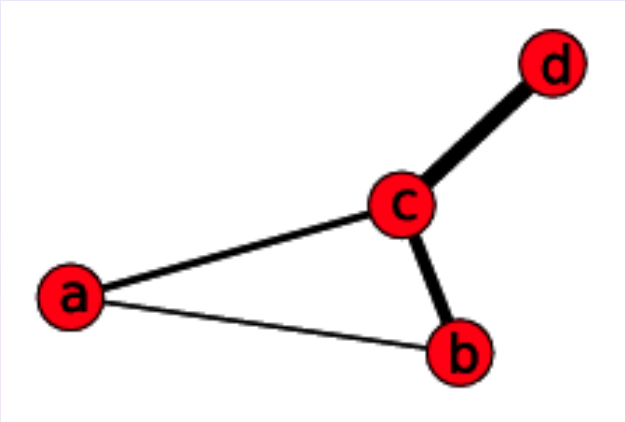

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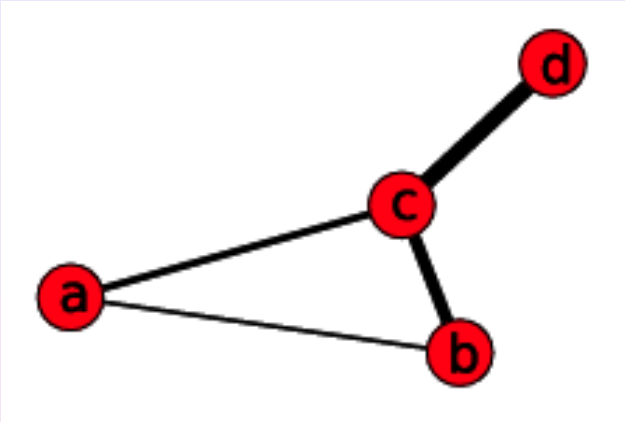

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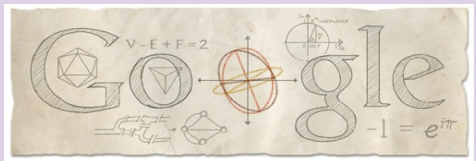
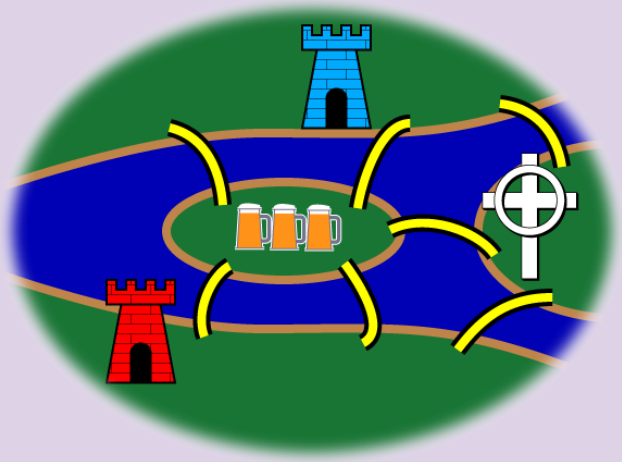
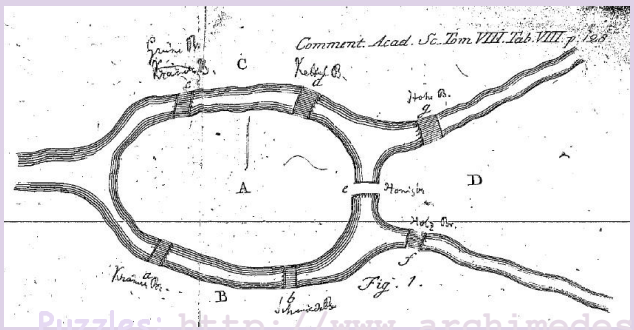
Network Topology

Graph Theory

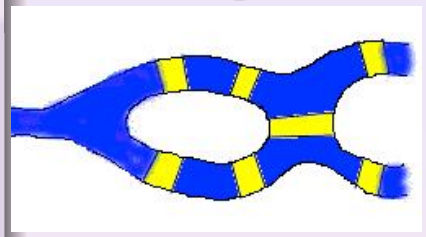
- 7 bridges of Könisgberg [Euler, 1736]

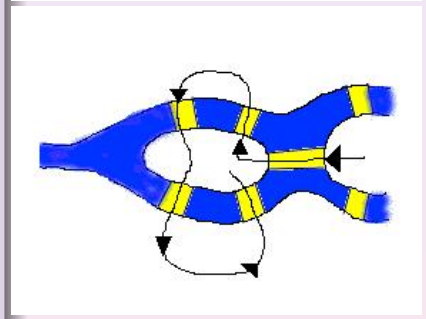


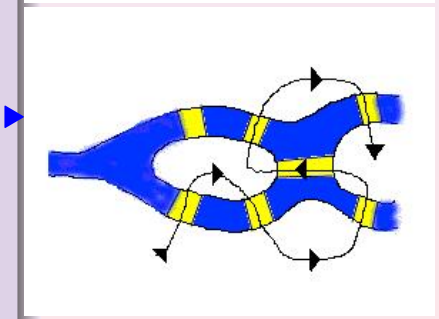




- 7 bridges







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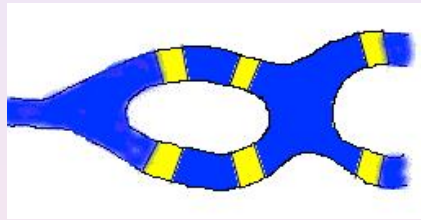

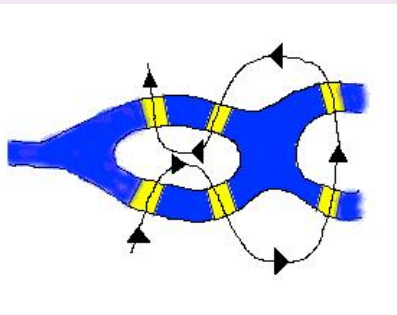

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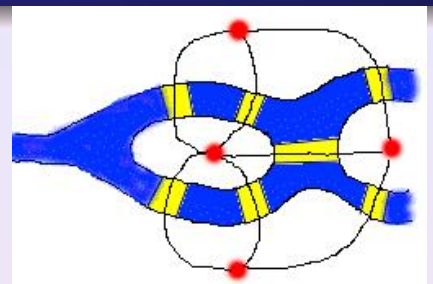

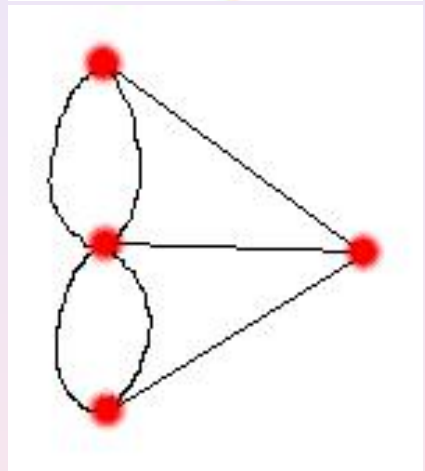

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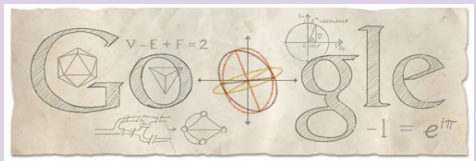

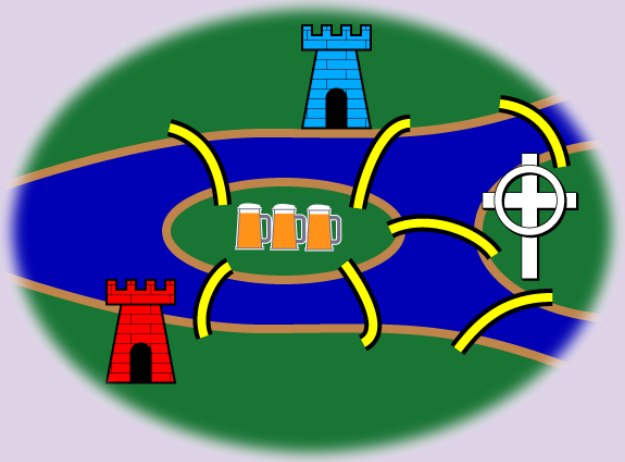

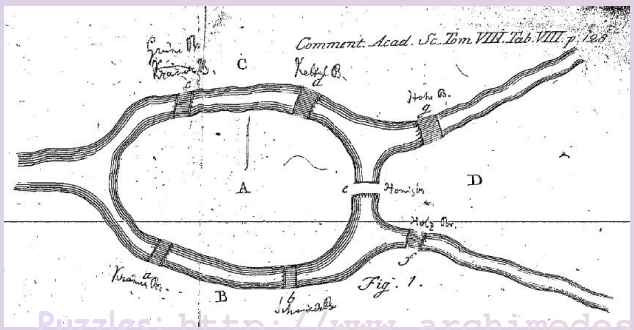

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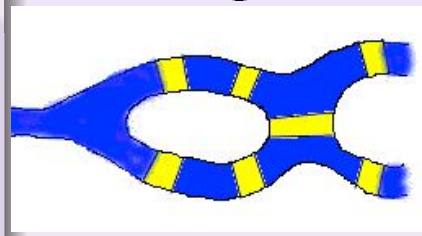

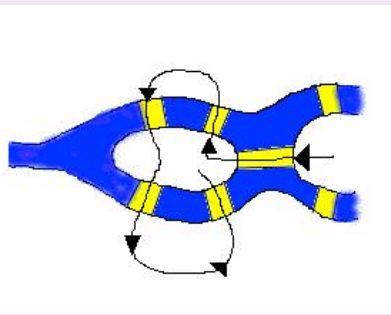

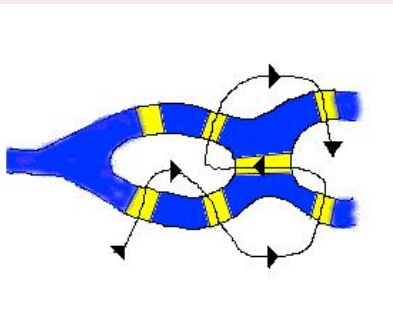

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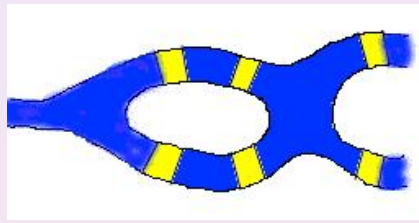

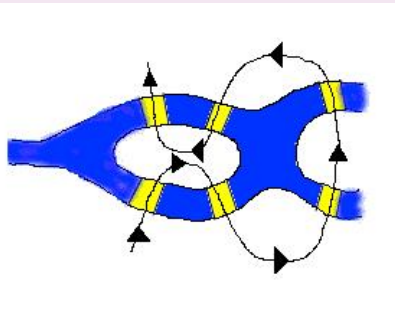

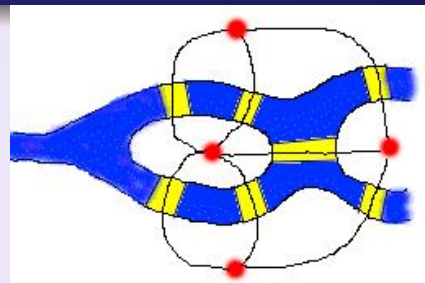

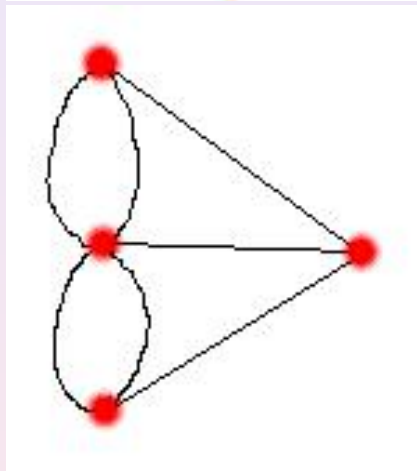

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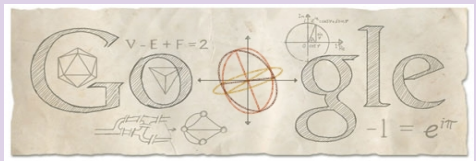

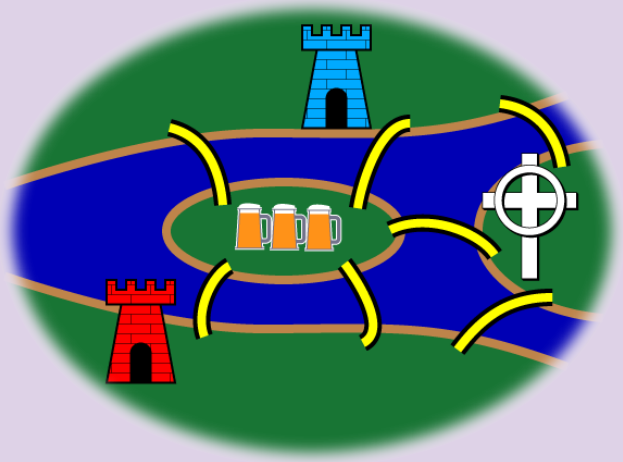

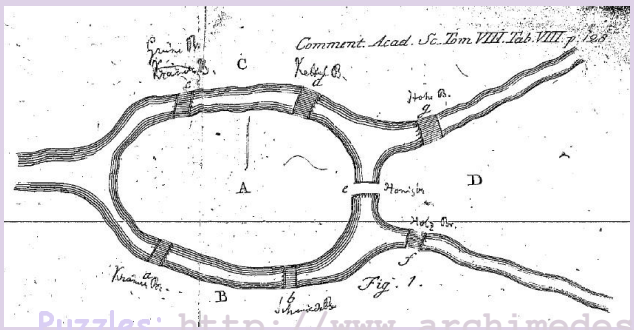

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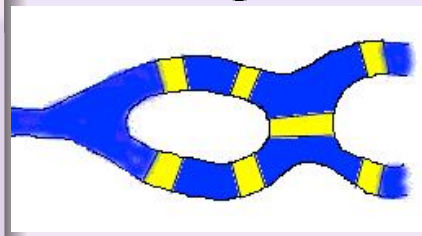

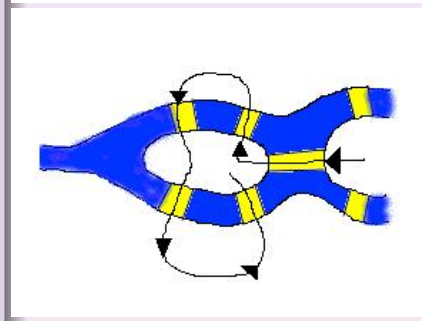

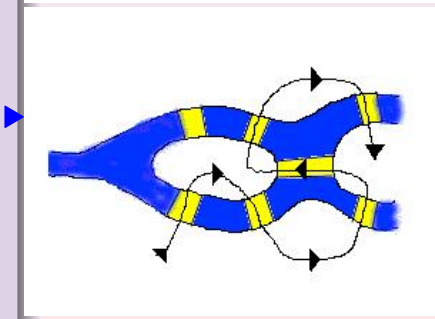

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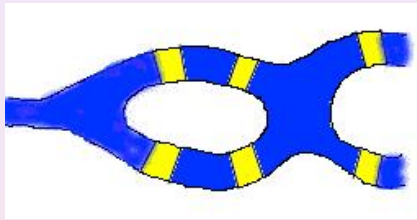

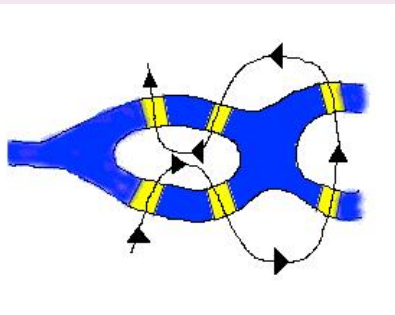

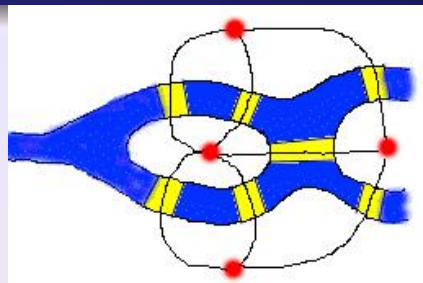

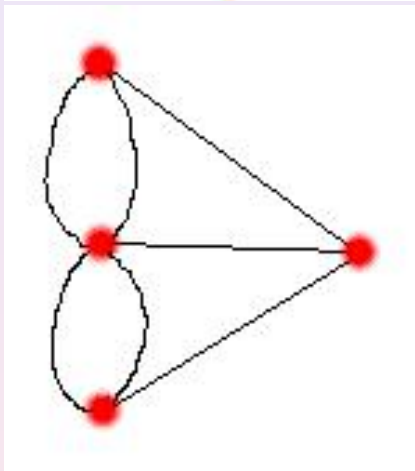

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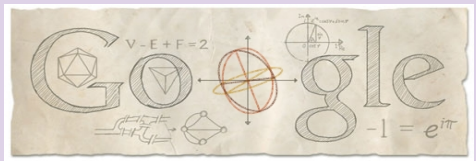

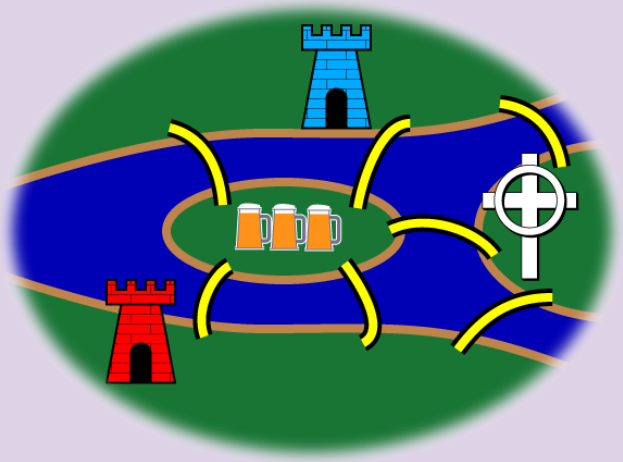

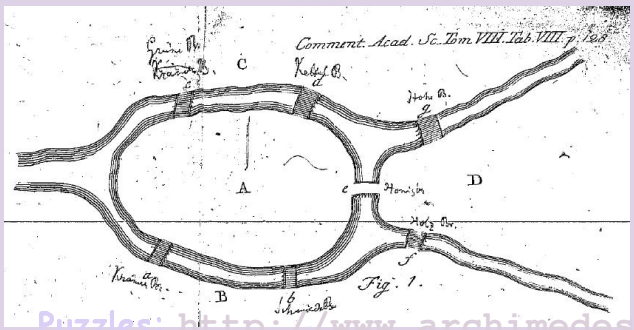

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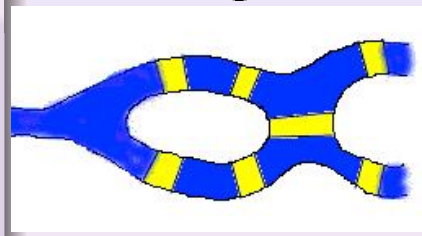

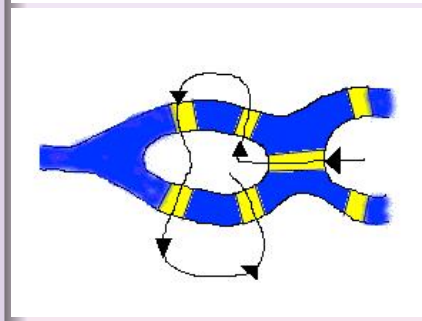

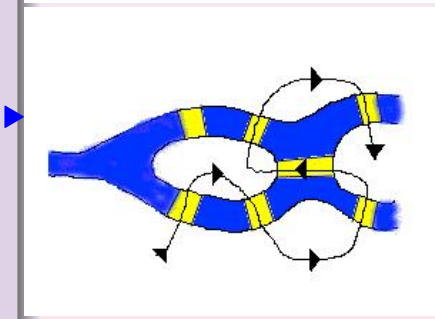

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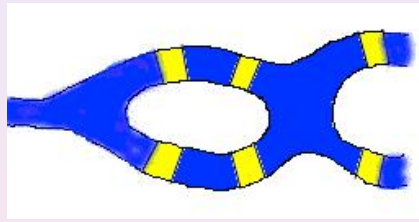

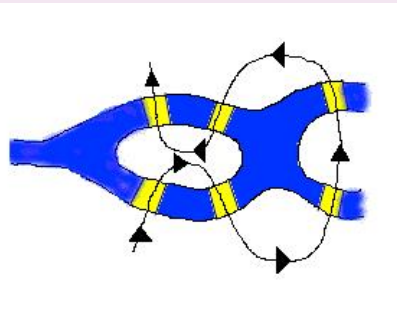

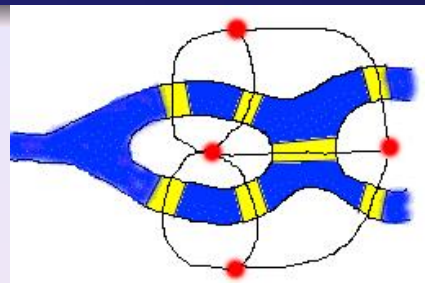

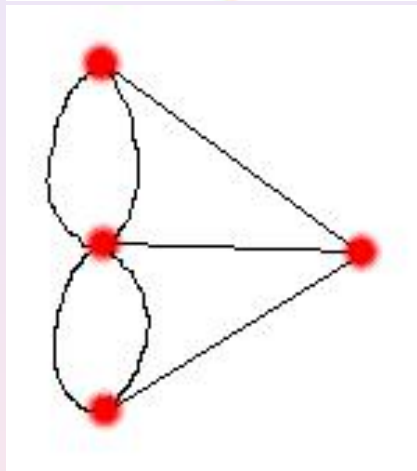

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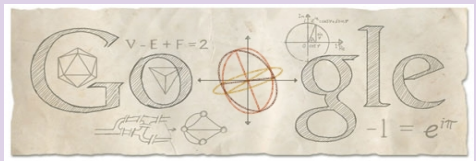

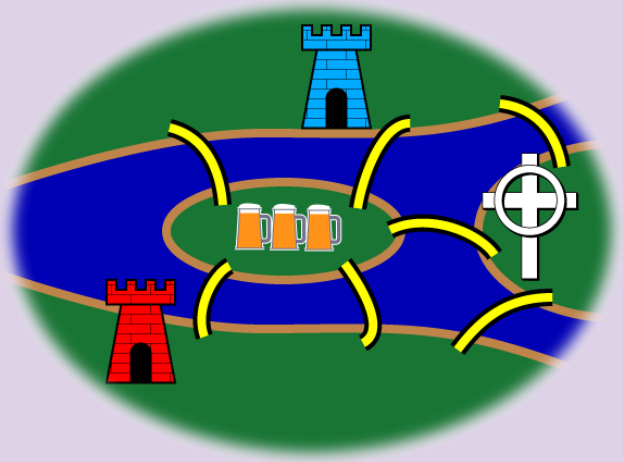

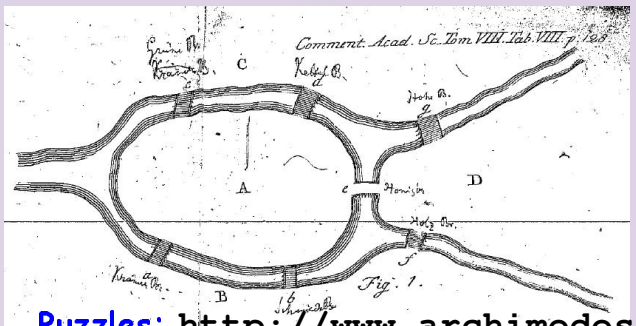

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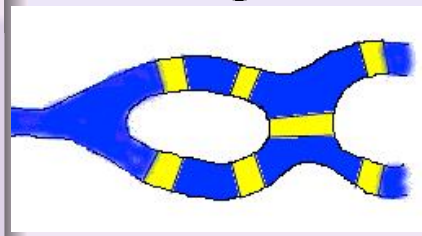

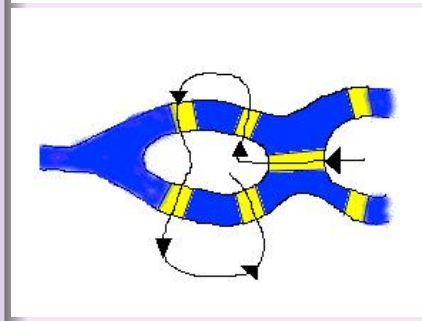

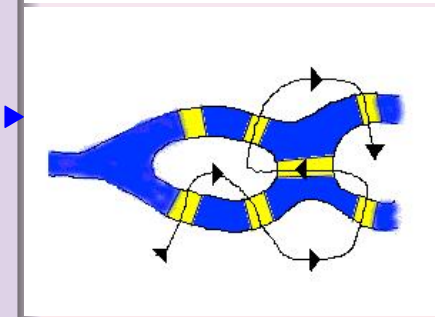

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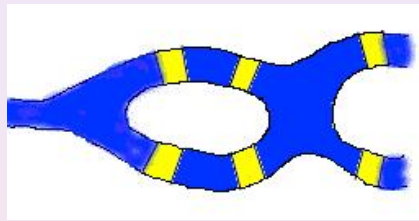

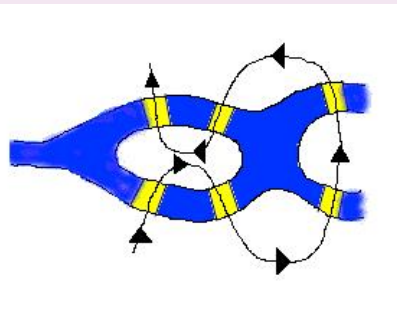

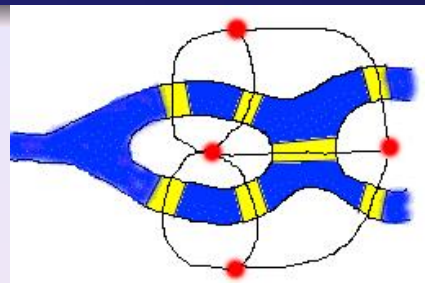

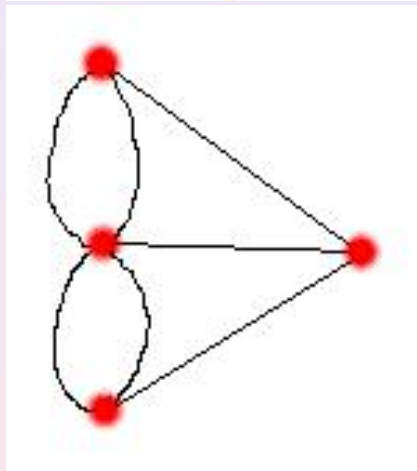

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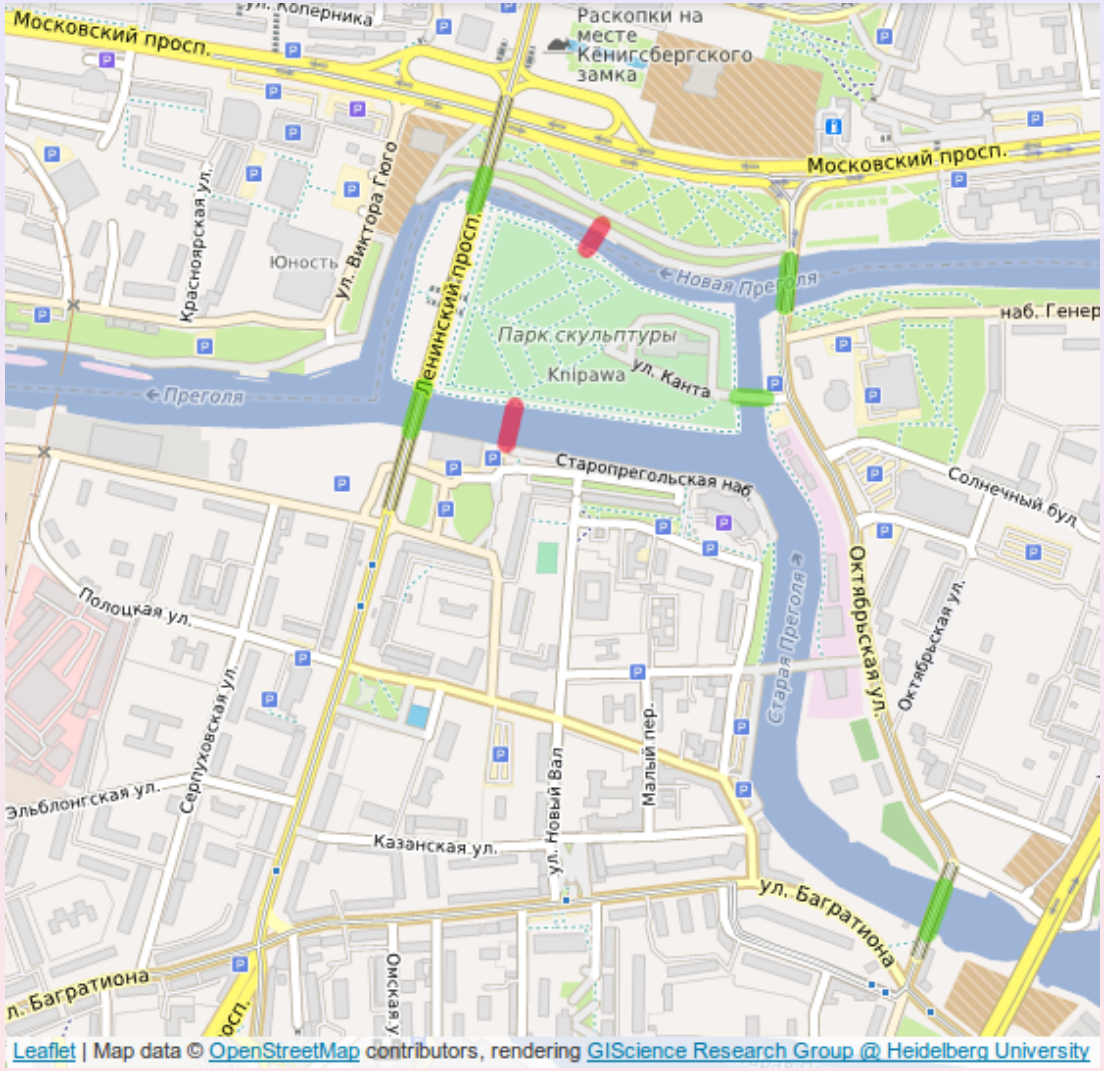
Present state of the seven bridges of Königsberg



Src: `http://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg`

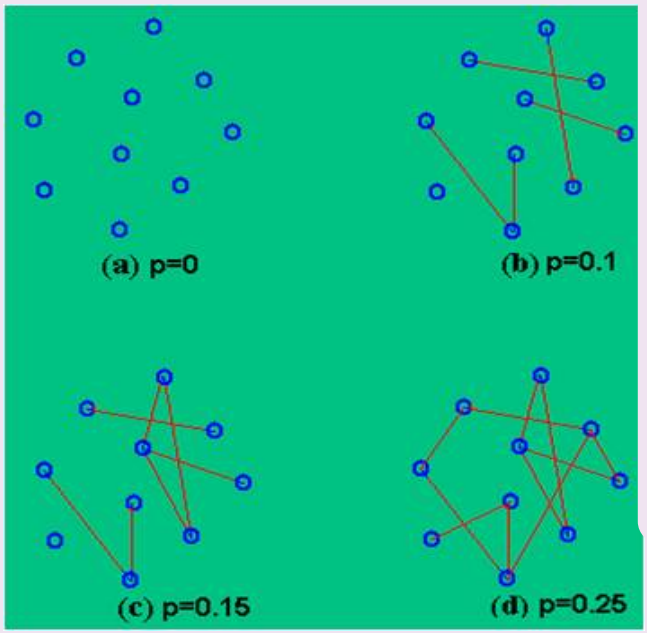
Network Topology

how nodes are connected

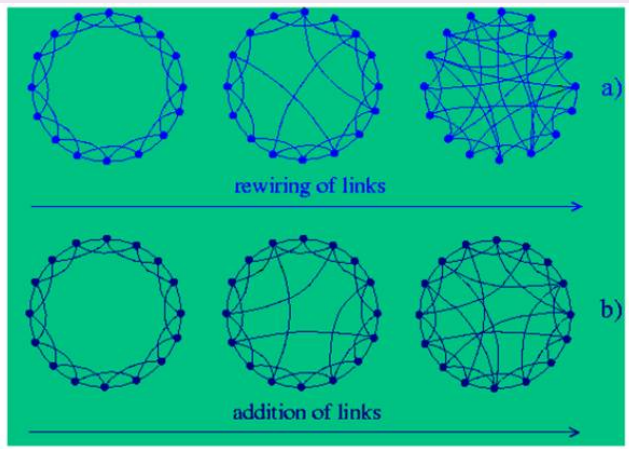
- According to the topology is possible to "classify" the networks.

Distribution of links $P(k)$ (degree distrib), avg. nbr. of links $\langle k \rangle$, clustering coeff, avg. length $\langle \ell \rangle$ (diam),...

- random networks
  - Erdös-Rényi (1960)



- regular or uniform
  - globally coupling, nearest-neighbors, ...



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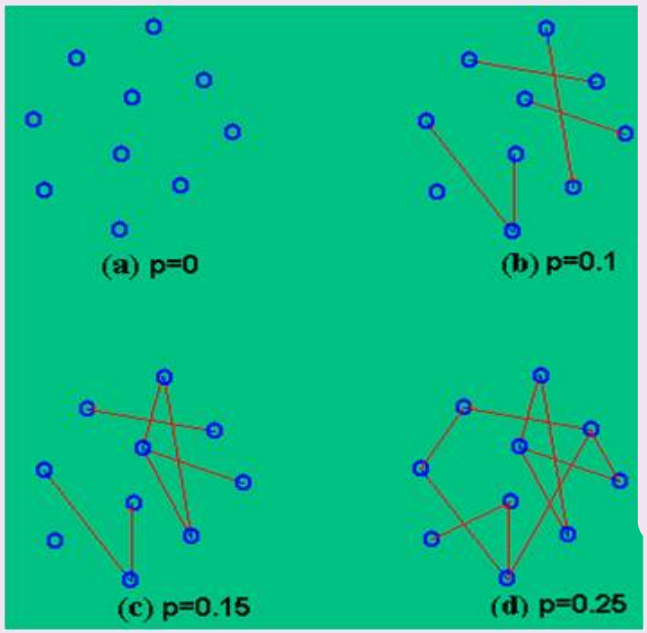

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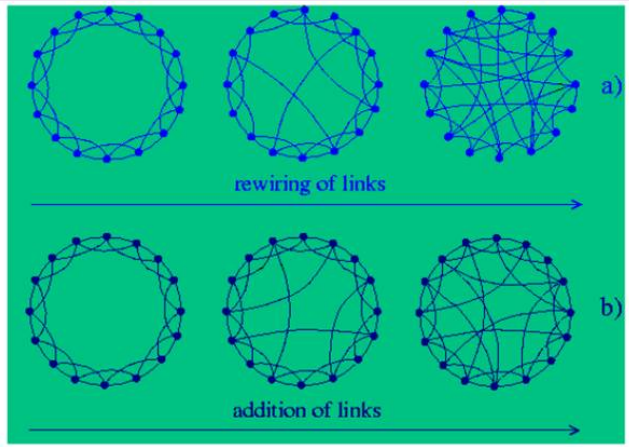

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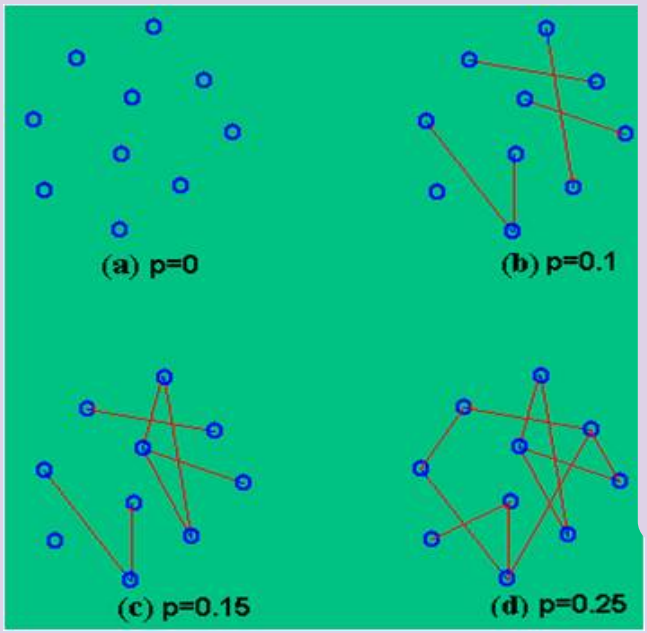

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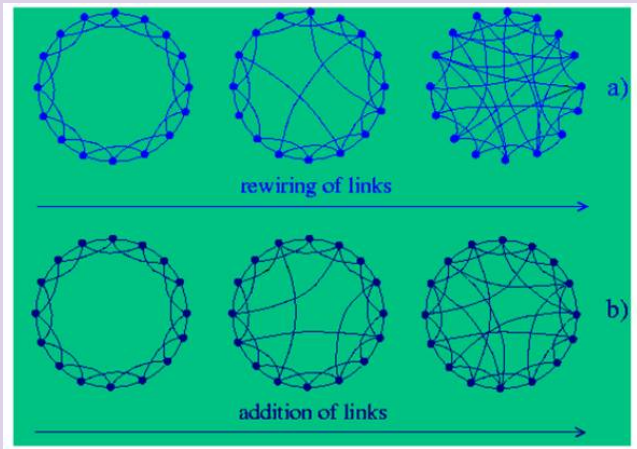

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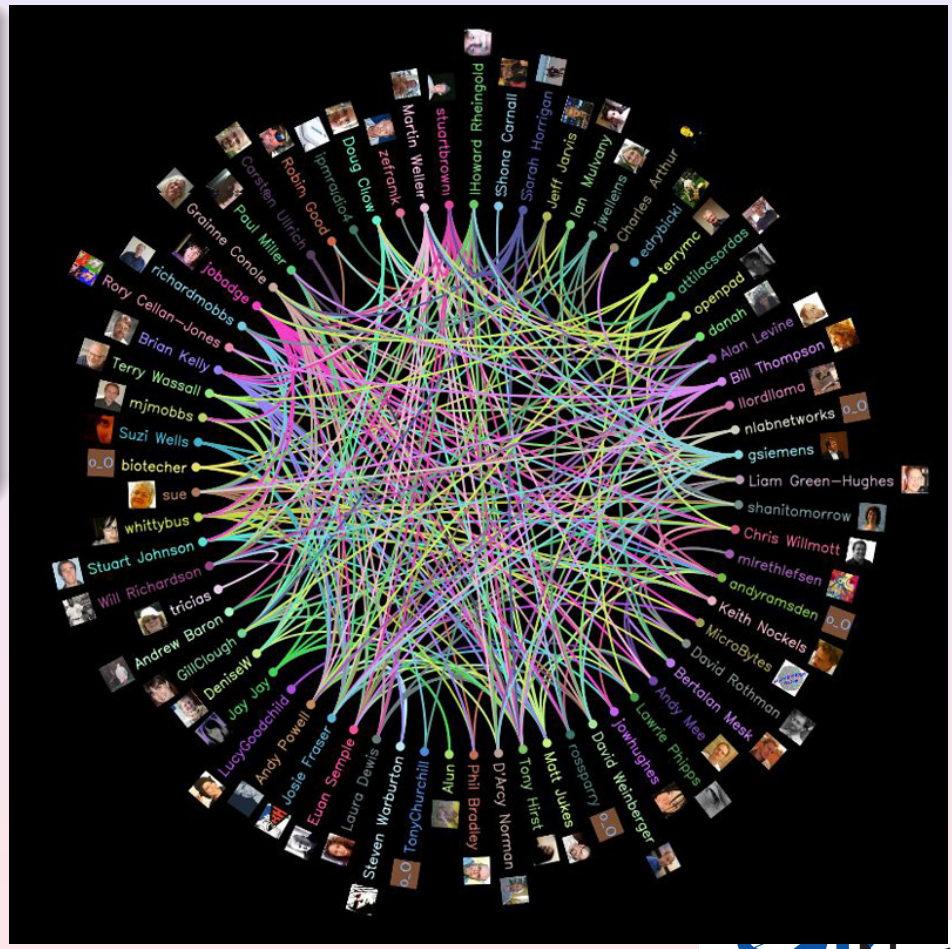
Complex Networks I

Properties of «REAL» networks

- **Small-World** [Watts and Strogatz, Nature, 1998]

most nodes can be reached by a small number of steps («6 dof»!)

- Connectivity distribution: uniform but decays exponentially
- Homogeneous nature: each node has roughly the same number of links



SciNet compute • calcul CANADA

Complex Networks II

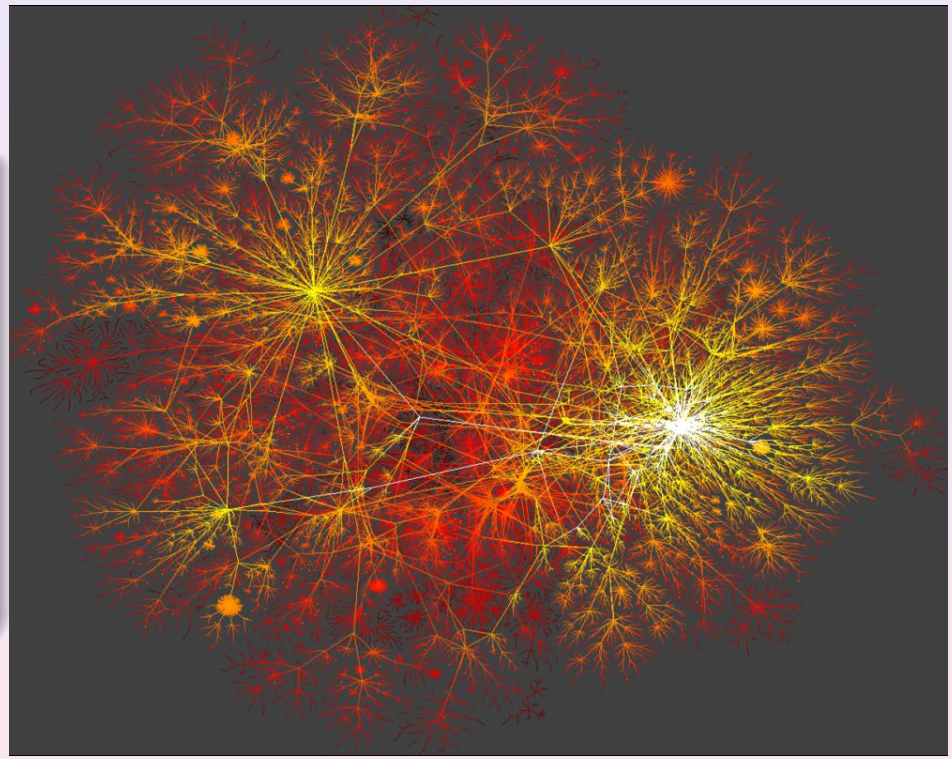
Properties of «REAL» networks

- **Scale-Free** [Barabási and Albert, Science, 1999]

some nodes act as "highly connected hubs", and most nodes are of low degree distribution:

$$P(k) = k^{-\gamma}$$

$$2 \leqslant \gamma \leqslant 3 \quad \text{(power law)}$$



MultiGraph: multiple edges

```
mg = nx.MultiGraph()
mg.add_weighted_edges_from([(1,2, .5), (1,2, .75), (2,3, .5)])
mg.degree()
{1: 2, 2: 3, 3: 1}
mg.degree(weight='weight')
{1: 1.25, 2: 1.75, 3: 0.5}
```

DiGraph: directed graphs

```
dg = nx.DiGraph()
dg.add_weighted_edges_from([(1,4,0.5), (3,1,0.75)])
dg.degree(1,weight='weight')
1.25
dg.successors(1)
[4]
dg.predecessors(1)
[3]
```

More graphs generators

```
K_5=nx.complete_graph(5)
nx.draw(K_5)

K_3_5=nx.complete_bipartite_graph(3,5)
nx.draw(K_3_5)

barbell=nx.barbell_graph(10,10)
nx.draw(barbell)

lollipop=nx.lollipop_graph(10,20)
nx.draw(lollipop)
```

Random graphs generators

```
er=nx.erdos_renyi_graph(100,0.15)
nx.draw(er)

ws=nx.watts_strogatz_graph(30,3,0.1)
nx.draw(ws)

ba=nx.barabasi_albert_graph(100,5)
nx.draw(ba)

red=nx.random_lobster(100,0.9,0.9)
nx.draw(red)
```

MultiGraph: multiple edges

```
mg = nx.MultiGraph()
mg.add_weighted_edges_from([(1,2, .5), (1,2, .75), (2,3, .5)])
mg.degree()
{1: 2, 2: 3, 3: 1}
mg.degree(weight='weight')
{1: 1.25, 2: 1.75, 3: 0.5}
```

DiGraph: directed graphs

```
dg = nx.DiGraph()
dg.add_weighted_edges_from([(1,4,0.5), (3,1,0.75)])
dg.degree(1,weight='weight')
1.25
dg.successors(1)
[4]
dg.predecessors(1)
[3]
```

More graphs generators

```
K_5=nx.complete_graph(5)
nx.draw(K_5)

K_3_5=nx.complete_bipartite_graph(3,5)
nx.draw(K_3_5)

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nx.draw(ws)

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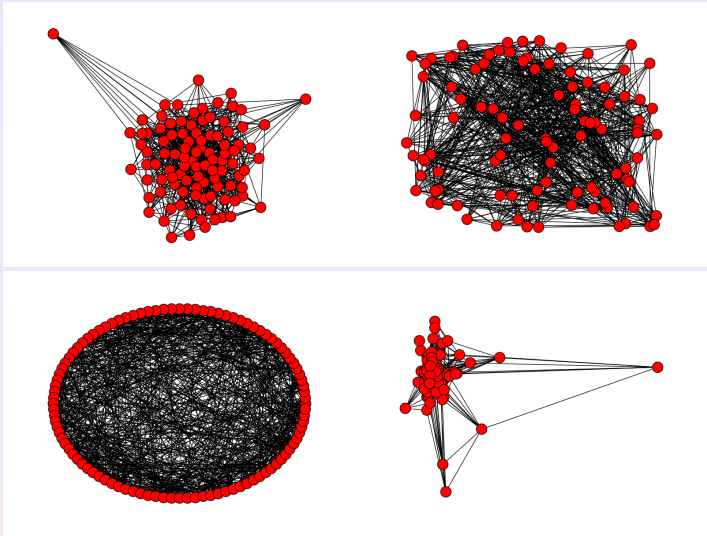
nx.draw(ba)

red=nx.random_lobster(100,0.9,0.9)
nx.draw(red)
```

Visualization & Graph properties...

Visualization

```
import networkx as nx
import pylab as plt
g = nx.erdos_renyi_graph(100,0.15)
nx.draw(g)
nx.draw_random(g)
nx.draw_circular(g)
nx.draw_spectral(g)
plt.savefig('graph.png')
```



Graph properties

```
N,K = g.order(), g.size()
avg_deg = float(K)/N
print "Nodes: ", N
print "Edges: ", K
print "Average degree: ", avg_deg
```

Degree distribution

```
g.in_degrees()
```

```
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
AttributeError: 'Graph' object has no attribute 'in_degrees'
```

Visualization & Graph properties...

Visualization

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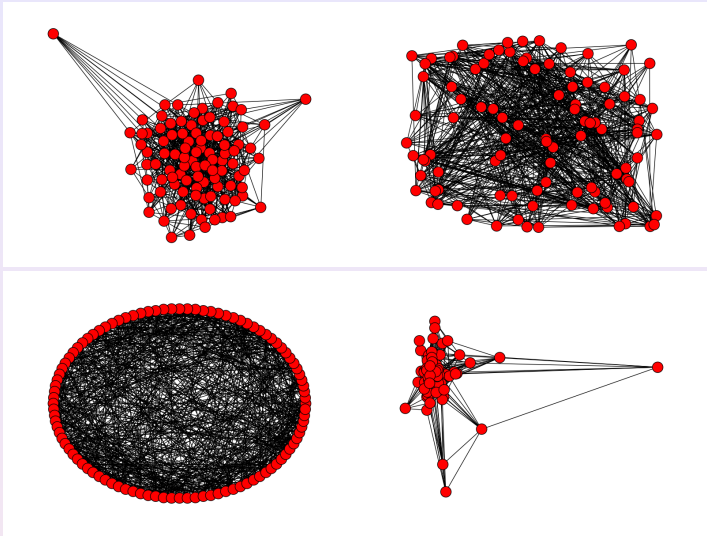

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Visualization & Graph properties...

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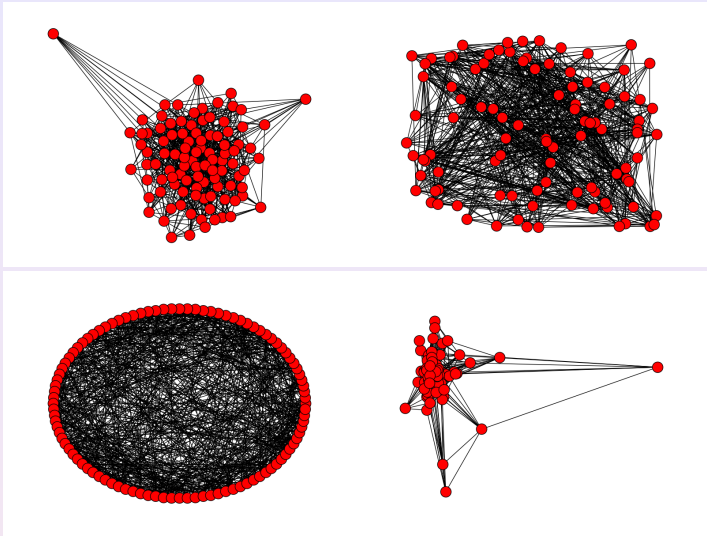

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Visualization & Graph properties...

Visualization

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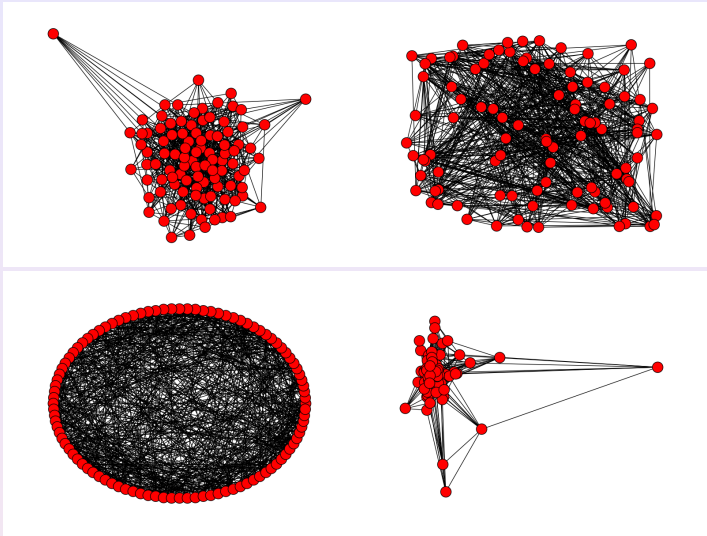
```



Graph properties

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print "Nodes: ", N
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print "Average degree: ", avg_deg
```

Degree distribution

```
g.in_degrees()
```

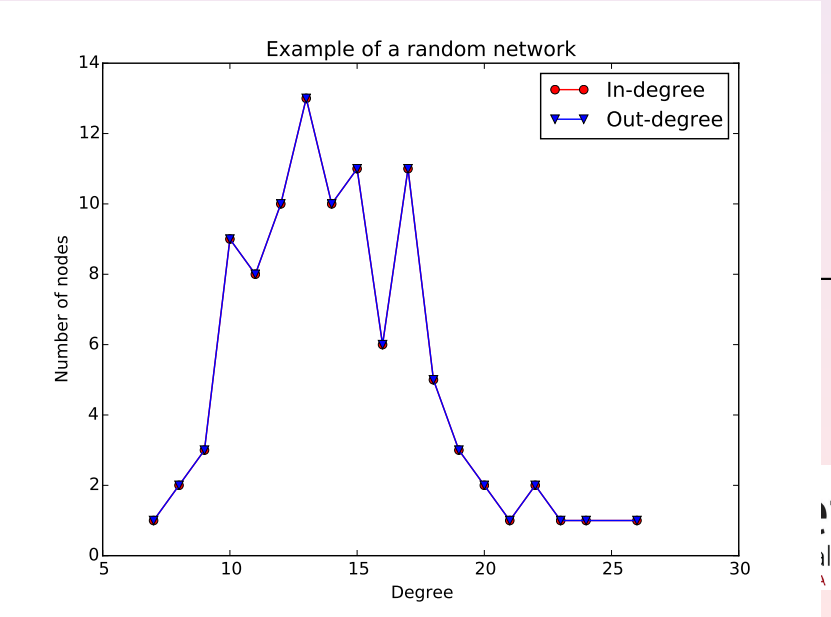
```
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
AttributeError: 'Graph' object has no attribute 'in_degrees'
```

Graph properties: degree distribution ...

Degree distribution

```
diG=nx.DiGraph(g) #create a directed graph
# account for in/out-going connections...
in_degrees = diG.in_degree() # dictionary node:degree
out_degrees = diG.out_degree() # dictionary node:degree
in_values = sorted(set(in_degrees.values()))
out_values = sorted(set(out_degrees.values()))
# put data in histogram-form
in_hist = [in_degrees.values().count(x) for x in in_values]
out_hist = [out_degrees.values().count(x) for x in out_values]

# plot
plt.figure()
plt.plot(in_values,in_hist,'ro-') # in-degree
plt.plot(out_values,out_hist,'bv-') # out-degree
plt.legend(['In-degree','Out-degree'])
plt.xlabel('Degree')
plt.ylabel('Number␣of␣nodes')
plt.title('Example␣of␣a␣random␣network')
plt.savefig('network_degree_distribution.pdf')
plt.close()
```

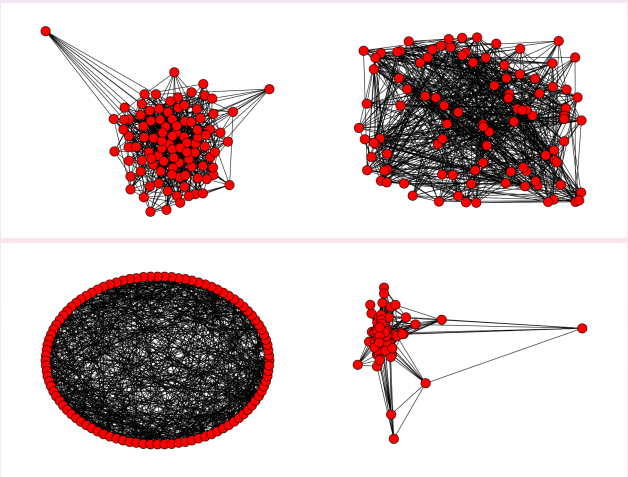


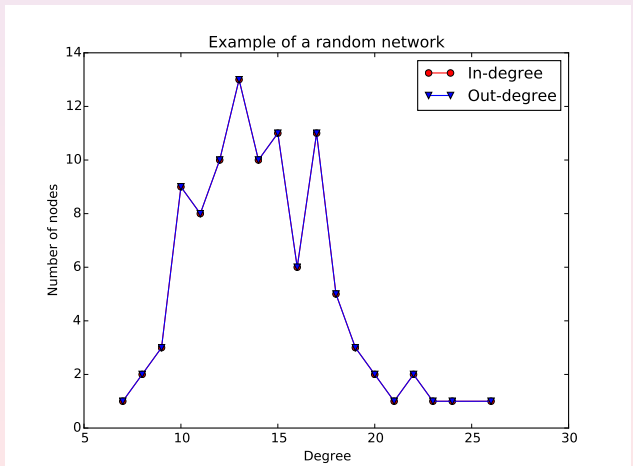
Graph properties: connectivity properties ...

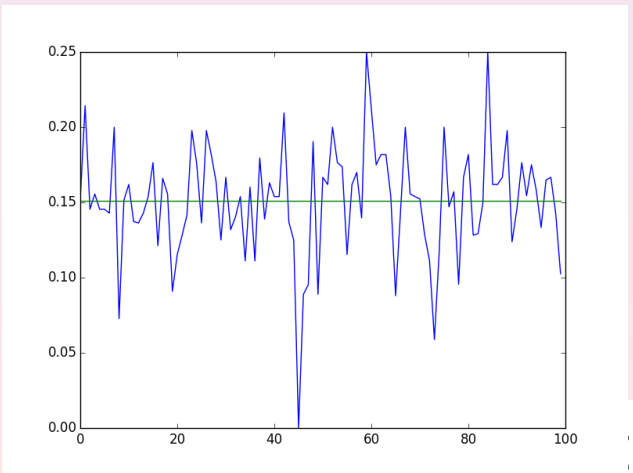
Clustering Coefficient

```
import numpy as np
g_ud = diG.to_undirected()
# Clustering coefficient of node 0
print nx.clustering(g_ud, 0)
# Clustering coefficient of all nodes (in a dictionary)
clust_coefficients = nx.clustering(g_ud)
# Average clustering coefficient
ccs = nx.clustering(g_ud)
avg_clust = sum(ccs.values()) / len(ccs)

plt.plot(ccs.keys(),ccs.values())
plt.plot(ccs.keys(),np.ones(len(ccs))*avg_clust)
```







compute • calcul CANADA

Modeling Twitter Relationships I

"Following"

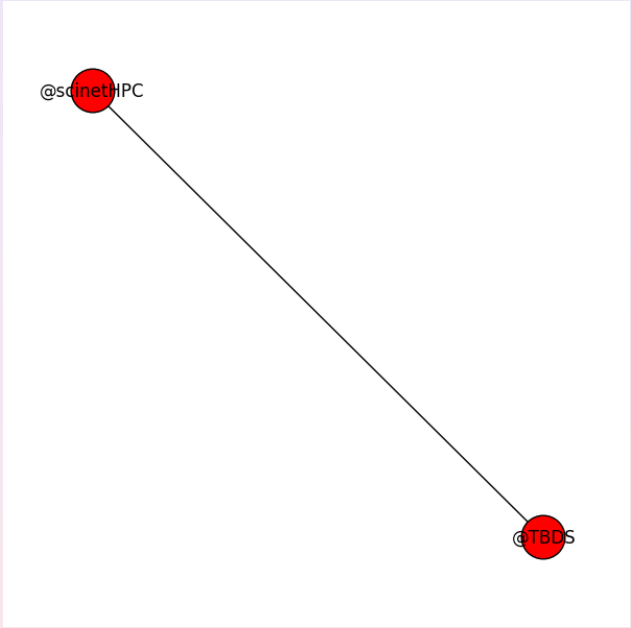
```
import networkx as nx
import pylab as plt

g = nx.Graph()  #nx.DiGraph()
g.add_node(0, {"handle": "@scinetHPC"})
g.add_node(1, {"handle": "@TBDS"})
g.add_edge(0, 1)

node_labels = nx.get_node_attributes(g,'handle')

plt.figure(figsize=(6,6))
plt.ion()

pos = nx.spring_layout(g)
nx.draw(g, pos, arrows=True, node_size=900)
nx.draw_networkx_labels(g, pos, labels = node_labels)
```



Modeling Twitter Relationships II

"Re-Tweets"

```
g = nx.DiGraph()
g.add_node(0, {"id": "@scinetHPC", "color": "r"})
g.add_node(1, {"id": "@TBDS", "color": "r"})
g.add_node(2, {"id": "Tweet␣#530038648860065793", "color": "orange"})
g.add_edge(0, 1, {"type": "follows"})
g.add_edge(0, 2, {"type": "tweeted"})

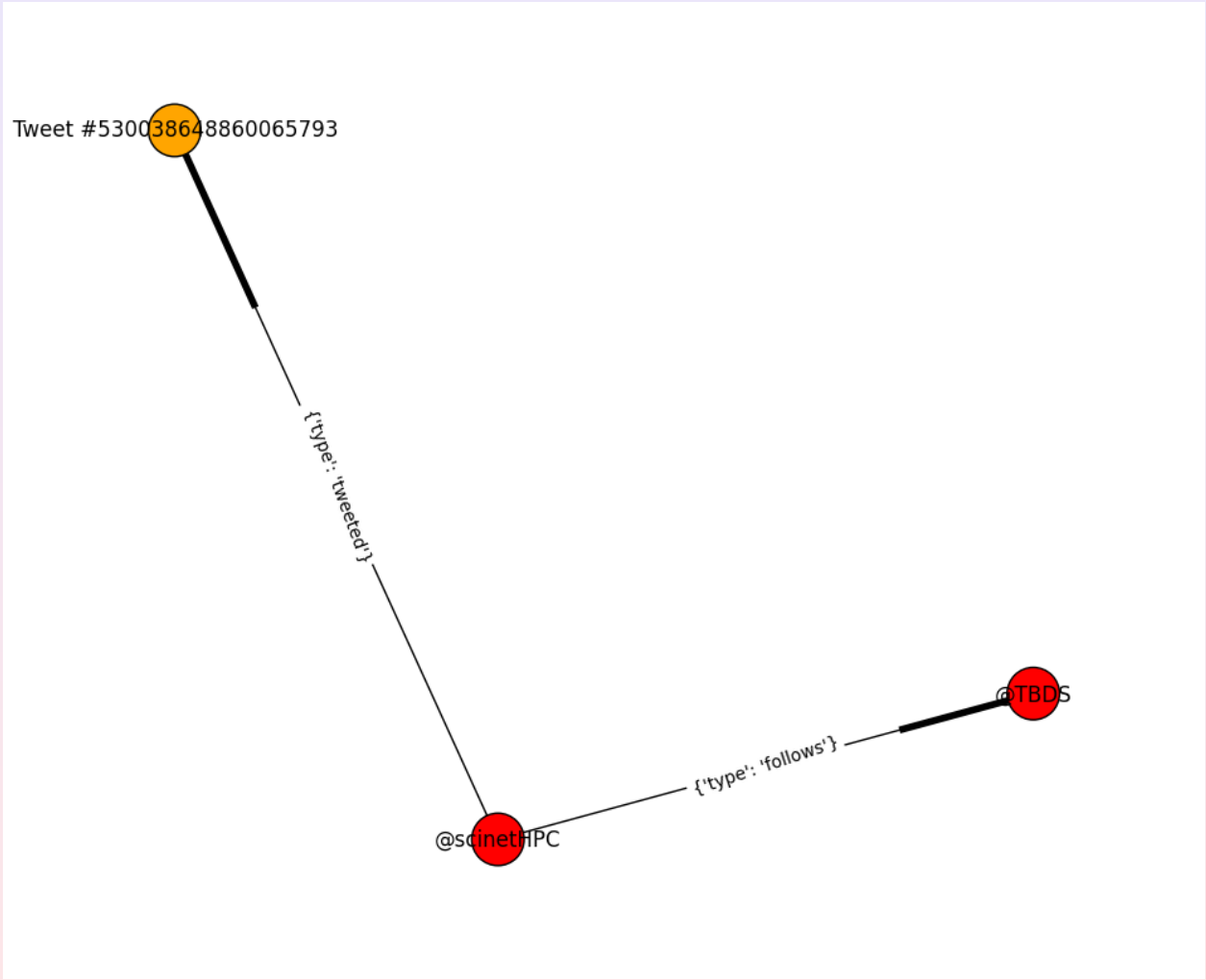
node_labels = nx.get_node_attributes(g,'id')
edge_labels = nx.get_edge_attributes(g,'type')
colors = nx.get_node_attributes(g, 'color')

plt.figure(figsize=(6,6))

pos = nx.spring_layout(g)
nx.draw(g, pos, arrows=True, node_size=900, node_color=colors.values())
nx.draw_networkx_labels(g, pos, labels = node_labels)
nx.draw_networkx_edge_labels(g, pos, labels = edge_labels)
```

Modeling Twitter Relationships III

A model for the Twitter network



Modeling the I/O-ops @ SciNet I

Data

```
tcs-f06n01   tcs-f06n01   0.013554078
tcs-f06n01   tcs-f06n01   0.004655432
tcs-f07n02   gpc-f119n017   0.002597279
tcs-f07n02   tcs-f07n02   0.000283613
tcs-f06n02   tcs-f06n02   0.000703586
tcs-f06n02   tcs-f06n02   0.001286946
tcs-f07n01   tcs-f07n01   0.025607990
tcs-f07n01   tcs-f07n01   0.018086633
tcs-f07n01   tcs-f07n01   0.003623529
tcs-f06n04   tcs-f06n04   0.012281074
tcs-f06n04   tcs-f06n04   0.012600201
tcs-f06n03   tcs-f06n03   0.006405814
tcs-f06n03   tcs-f06n03   0.024002783
tcs-f06n03   tcs-f06n03   0.019571064
tcs-f07n04   tcs-f07n04   0.034180323
tcs-f07n04   tcs-f07n04   0.100195957
```

449 records

networkX.read_edgelist(...)

```
g =

nx.read_format("path/to/file.txt",...options..

nx.write_format(g,"path/to/file.txt",...option
```

Read and write edge lists

```
g = nx.read_edgelist(path,comments='#',
    create_using=None,
    delimiter=' ',nodetype=None,
    data=True,edgetype=None,
    encoding='utf-8')

nx.write_edgelist(g,path,comments='#',
    delimiter=' ',data=True,

    encoding='utf-8')
```

Modeling the I/O-ops @ SciNet II

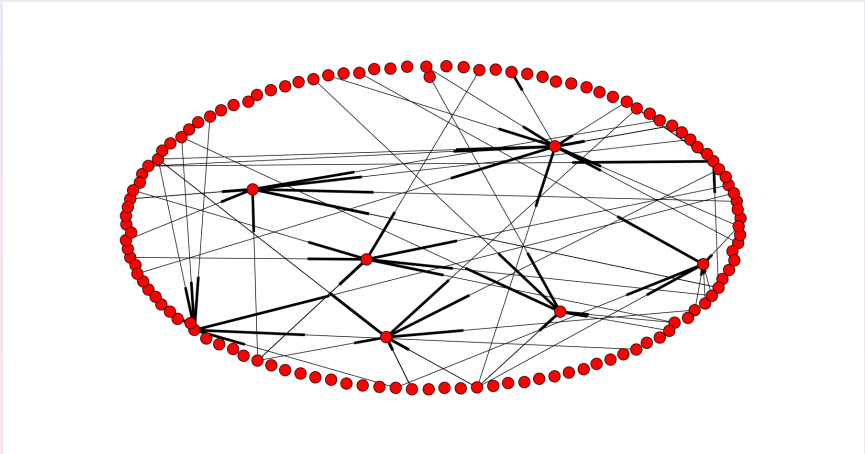
Importing and plotting the data

```
import networkx as nx
import pylab as plt

scinetG = nx.read_edgelist("scinet_data.txt",
        comments='#',
        create_using=nx.MultiDiGraph(),
        delimiter='␣␣',
        nodetype=str,data=(('weight',float),) )

plt.ion()
nx.draw(scinetG)


plt.figure()
```



Modeling the I/O-ops @ SciNet III

"Massaging" the network (data)

```
plt.figure()
plt.axis('off')

cutOff=0.001
elarge=[(u,v) for (u,v,d) in scinetG.edges(data=True) if d['weight'] > cutOff]
esmall=[(u,v) for (u,v,d) in scinetG.edges(data=True) if d['weight'] <= cutOff]

#try different ones from the ones below...
#pos=nx.spring_layout(scinetG) # positions for all nodes
pos=nx.graphviz_layout(scinetG)
#pos=nx.shell_layout(scinetG)
#pos=nx.random_layout(scinetG)
#pos=nx.spectral_layout(scinetG)

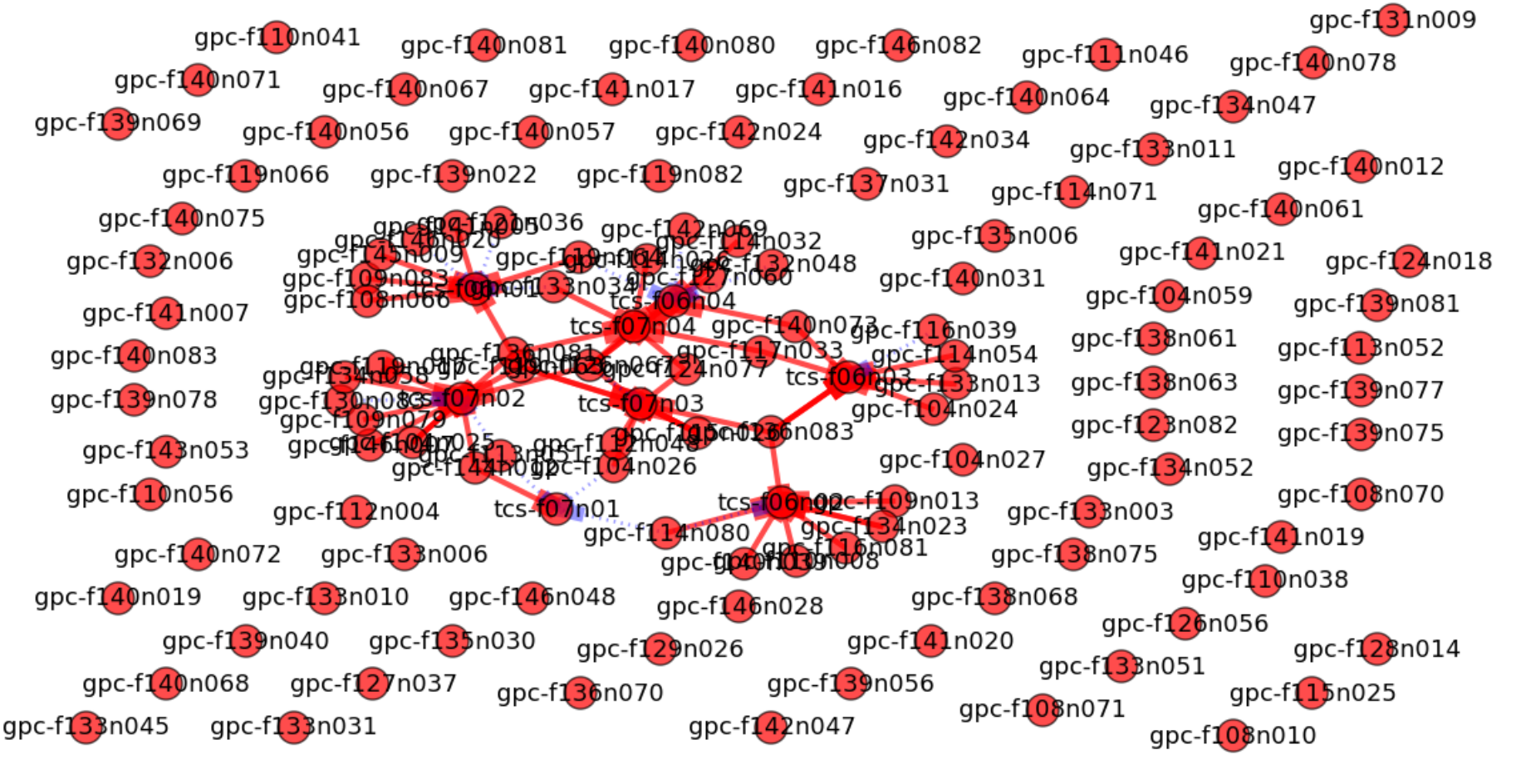
# nodes
nx.draw_networkx_nodes(scinetG,pos,
#                 node_color=nodeType.values(),
#                 node_size=tmW.values(),
                cmap=plt.cm.Reds, alpha=0.7)

# edges
color_edges=range(scinetG.number_of_edges())
nx.draw_networkx_edges(scinetG,pos,edgelist=elarge,
                    width=3,alpha=0.7,edge_color='r',
                    with_labels='True',edge_cmap=plt.cm.Blues)
nx.draw_networkx_edges(scinetG,pos,edgelist=esmall,
                    width=2,alpha=0.4,edge_color='b',style='dotted')
```

Modeling the I/O-ops @ SciNet IV

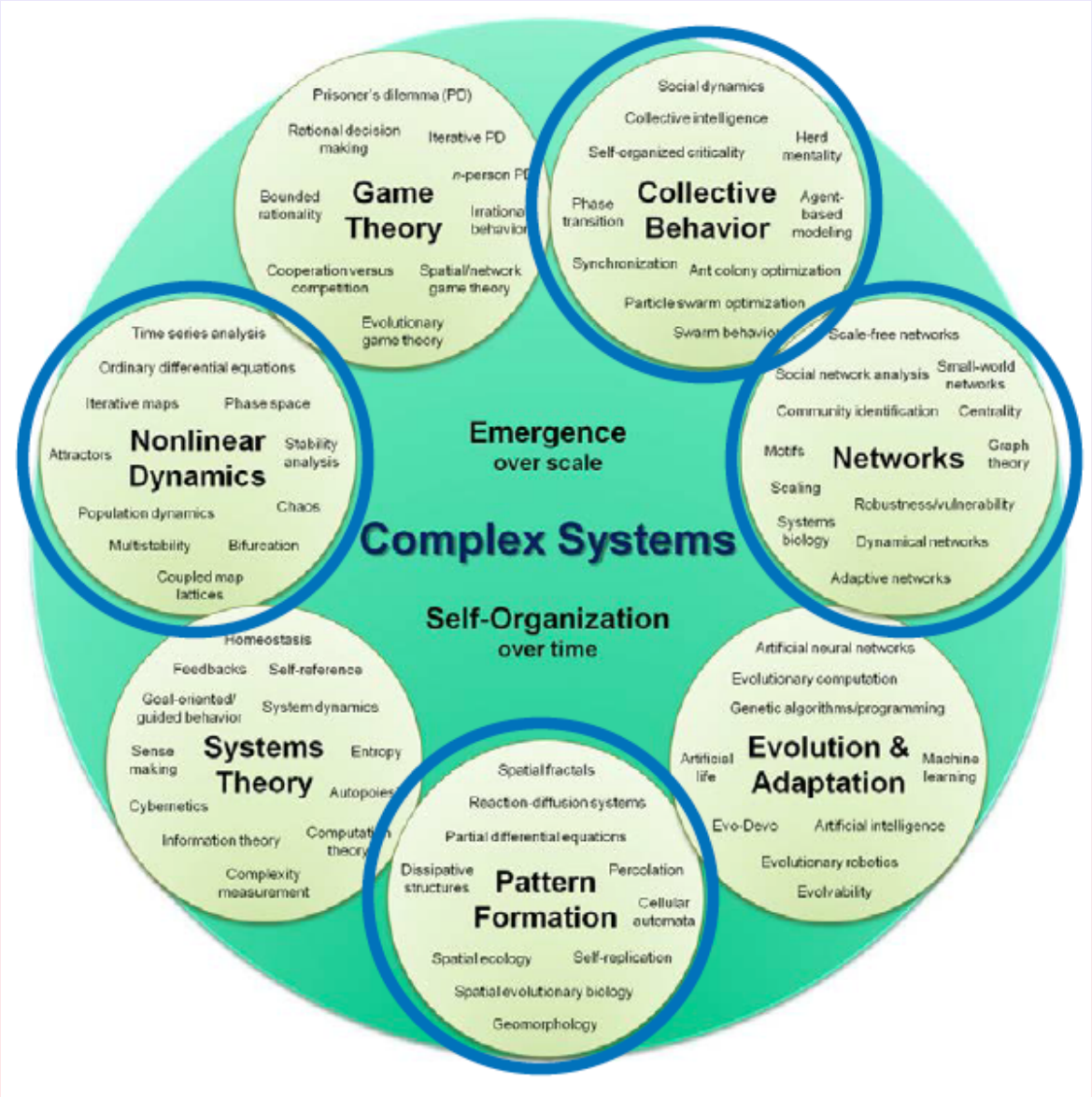
```
# labels
try:
        nx.draw_networkx_labels(scinetG,pos,labels,font_size=13,
                font_family='sans-serif', font_color='Indigo')
except:
        nx.draw_networkx_labels(scinetG,pos,font_size=13,font_family='sans-serif')
```

Modeling the I/O-ops @ SciNet V



PyCX: Python-based CompleX systems simulations

Online repository: **http://pycx.sf.net/**



Further Resources & Examples

- NetworkX: Network Analysis with Python, Salvatore Scellato
  http://www.cl.cam.ac.uk/~cm542/teaching/2011/stna-pdfs/stna-lecture11.pdf
- Graph Theory, including modelling of relationships, tweets, how to build a NetworkX graph from Twitter crawl results ...
http://nbviewer.ipython.org/github/jquacinella/IS609_Group/blob/master/GraphTheory/Graph%20Theory%20-%20Social%20Networking.ipynb
- Graph and Network Analysis
  http://mlg.ucd.ie/files/summer/tutorial.pdf
- Social Network Analysis in Python
  https://ep2013.europython.eu/media/conference/slides/social-network-analysis-in-python.pdf
- Tutorial on PyCX
  http://pycx.sourceforge.net/PyCX-ECAL2013-tutorial.pdf

More about Complex Networks

NECSI:

http://necsi.edu/ http://www.necsi.edu/publications/dcs/index.html

http://www.necsi.edu/events/vidlib/ http://www.necsi.edu/research/overview.php

Some other good references:

http://www-personal.umich.edu/~mejn/courses/2004/cscs535/review.pdf

http://www.ifr.ac.uk/netsci08/Download/Invited/ws1_Caldarelli.pdf

http://barabasilab.neu.edu/courses/phys5116/

SantaFe Institute:

http:

//www.santafe.edu/education/schools/complex-systems-summer-schools/2015-program-info/